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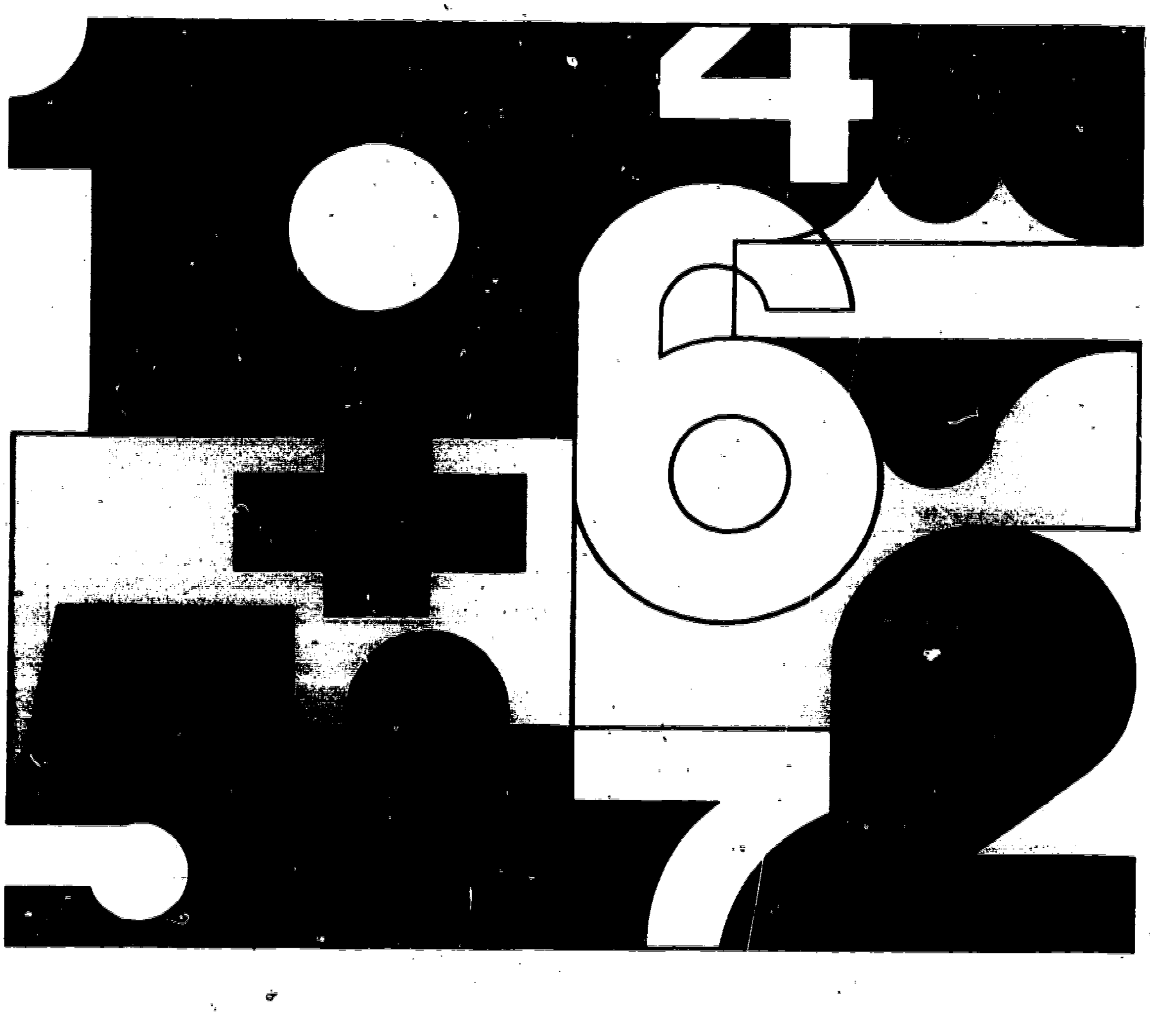
ABSTRACT

The results of three investigations into major influences affecting women's participation and achievement in mathematics and their preferences for mathematics-related careers are reported. The major cognitive, affective and educational influences are studied. Major social influences, including sex role stereotyping mathematics as a male domain, and perceived usefulness of mathematics for later life roles, are also discussed. Evidence related to hypotheses of biological sources as major influences of differences in mathematical achievement are investigated. The underlying theme of all three papers is the finding that sex differences in mathematics achievement, which are usually evident for the first time around grade 8 or 9, are largely a function of differential course taking rather than any inability on the part of women to learn mathematics. Mathematics tests that do show sex differences usually do not control for the number of courses taken. Also, differences do not appear to have a biological basis. (Author/MP)

# Women and Mathematics: Research Perspectives for Change

NIE Papers in Education and Work; No. 8







The National Institute of Education
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Washington, D.C. 20208

NIE Papers in Education and Work: Number Eight

# WOMEN AND MATHEMATICS:

# RESEARCH PERSPECTIVES FOR CHANGE

By

Lynn H. Fox, Johns Hopkins University
Elizabeth Fennema, University of Wisconsin
and
Julia Sherman, Women's Research Institute of Wisconsin

November, 1977

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2. Facilities Handbook for Career Education, By William B. Gill and Ann W. Luke of System Development Corporation.

3. Sex Discrimination in the Selection of School District Administrators: What Can be Done?, by Doris M. Timpano of Career Women in Education and Louise W. Knight.

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7. The Career Intern Program: Final Report (two volumes: Vol. I, An Experiment in Career Education That Worked; Vol. II, Technical Appendices.

8. Women and Mathematics: Research Perspectives for Change, by Lynn H. Fox, Elizabeth Fennema and Julia Sherman.

9. Industry/Education Community Councils (Available fall, 1977).

10. Sex-Fair Interest Measurement: Research and Implications, edited by Carol Kehr Tittle and Donald G. Zytowski (Available fall, 1977).

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FOREWORD

Why is NIE studying the area of women and mathematics?

The Education and Work Group of the National Institute of Education has developed a program in the area of women and mathematics as a strategy to increase the educational and occupational choices of women, which in comparison to the options chosen by men, are typically restricted to a narrow range of service and professional occupations, such as secretary, nurse, librarian, and teacher. For example, in 1973, 40% of all women workers were in ten occupations and about one-half were in occupations which were more than 70% female. Men's occupational choices are typically much more diverse. This concentration of women into a few narrow occupations is termed occupational segregation by sex, a problem that the Education and Work Group is seeking to reduce by several different strategies. The women and mathematics program is one such strategy.

Several researchers, notably Lucy Sells and Nancy Kreinberg, have identified mathematics as a critical skill directly related to educational and occupational choices. Mathematical competence is useful for admission to many colleges and most professional occupations, in both technical and non-technical fields. Many non-college occupations also demand mathematical competence. Most of the mathematics-related fields, such as engineering, statistics, computer science, physics, navigation and actuarial science tend to be male-dominated and high paying. Yet current enrollment figures show that fewer women than men choose to study mathematics beyond the minimal school requirements. Thus women preclude themselves from entering occupations requiring some mathematical competence, which contributes to further occupational segregation by sex. Differences between men and women in their study of mathematics persist despite the fact that current research indicates that there are no significant differences in their achievement in mathematics before the seventh or eighth grades.

How do the three papers in this volume contribute to the women and mathematics program?

In order to plan for research in the area of women and mathematics, NIE asked three researchers--Dr. Elizabeth Fennema, University of Wisconsin-Madison, Dr. Julia Sherman, Women's Research Institute, and Dr. Lynn Fox, Johns Hopkins University--to investigate what was known about the major influences affecting women's participation and achievement in mathematics and their preferences for mathematics-related careers. The results of their investigations are included in this volume. As can be seen in the brief biographical sketches of these researchers, also included in this volume, each researcher brought a unique perspective and set of talents to her task.

Their assignment was to investigate current research to locate major influences on girls' achievement and participation in mathematics. Why don't girls choose to study mathematics more often? What are the major personal, psychological and social influences affecting their choices? The three papers were based on three different perspectives. Fennema chose to study the major cognitive, affective and educational influences, relying extensively on her own current research which was sponsored by a grant from the National Science Foundation. Major social influences, including sex role stereotyping mathematics as a male domain, and perceived usefulness of mathematics for later life roles, was the subject of Fox who drew on her own experience with mathematically talented girls. Sherman investigated evidence related to hypotheses of biological sources as major influencers of differences in mathematics achievement. Each of these papers included scholarly, analytical reviews of the most current research available accompanied by a detailed annotated bibliography, also included in this volume. Based on their reviews and their own unique experiences, each author wove existing information into recommendations for new directions of research in the area of women and mathematics.

These three papers have served as the foundation for planning the women and mathematics program. They served as the focal point for a national planning conference in area of women and mathematics. Both the papers and the conference contributed heavily to the development of a research grants program for fiscal years 1978 and 1979. The papers have already been disseminated to hundreds of interested people who have heard

of the women and mathematics program through a recent article in Time Magazine (March 14, 1977), through presentations at national conventions, and through NIE's own announcements of funding opportunities.

What did we learn from these three papers?

The underlying theme of all three papers is the finding that sex differences in mathematics achievement, which are usually evident for the first time around grade 8 or 9, are largely a function of differential course taking rather than any inability on the part of women to learn mathematics. Mathematics tests that do show sex differences usually do not control for the number of courses taken so that when 17-year-old boys are compared to 17-year-old girls the comparison is actually between students with 3-4 years of mathematics and those with 1-2 years of mathematics. Fennema contends that if girls' enrollment in mathematics could be increased, most of the differences in achievement could be eliminated. On this topic, Fox remarks that "Although sex differences in course-taking may not explain all the observed differences on tests of achievement and aptitude, course-taking clearly contributes heavily to sex differences in both achievement and adult career options."

Sex differences in mathematics achievement, when they do occur, do not appear to be linked to biological differences, indicates Sherman. She proposes one hypothesis after another--sex related differences in level of serum urate, effects of estrogens, X-linked characteristics, inheritance of spatial visualization and problem-solving abilities, earlier left cerebral dominance in females--and examines the evidence under each hypothesis with a critical eye. She determines that for most hypotheses, the evidence was weak or non-existant. Only one hypothesis--earlier left cerebral dominance in females--had much supporting evidence. Sherman explains that this early reliance on left cerebral functions which are linked to verbal, analytical modes of learning could lead to a preference or reliance on verbal learning rather than spatial-gestalt learning associated with the right half of the brain. Such a preference for verbal learning might hamper mathematical learning, which has a strong spatial-visual component. At the same time, Sherman suggests that spatial learning might be enhanced by training and she suggests further research in this area.

Fennema also notes the importance of the ability to visualize spatially as one of the most salient cognitive factors related to sex differences in mathematics achievement. She notes that current evidence suggests male superiority on tests of spatial visualization, beginning during adolescence, the same time when differences in mathematics achievement begin to emerge. Fennema also notes that current attitudes and values of students, their parents and teachers may also affect women's participation and achievement in mathematics. One important attitude is the stereotyping of mathematics as a male domain. For example, boys are expected to perform better than girls in mathematics and fathers are consulted more often than mothers for help with homework in mathematics. Because many girls perceive mathematics as a male subject, they are less motivated to excel in mathematics. Achievement in mathematics is generally not expected nor reinforced for girls. Another important attitude that shows some relation to sex differences in mathematics is confidence in one's ability to achieve. When boys fail in mathematics they say "I didn't try hard enough." When girls fail they say "Im not good at math." Sex differences also occur in the perceived usefulness of mathematics to future adult roles. Girls don't see the usefulness of mathematics to their future roles as wives and mothers. Boys are more likely to see its usefulness for future careers. Characteristics of teachers, instruction and school organization also play a role in women's participation and achievement in mathematics, as Fennema explains in her paper.

Fox summarizes the major social influences related to sex differences in mathematics achievement. In addition to some of the attitude differences described in Fennema's paper, Fox notes a difference between boys' and girls' career orientation. Girls more often than boys perceive a role conflict between family and work roles; girls are less career-oriented than boys. Girls seem to be less interested in math-related careers, and they are less knowledgeable than boys about the relevance of mathematics to a variety of careers. The most important people influencing girls' participation in mathematics are parents; less influential are teachers and counselors. Parents tend to sex-type mathematics and are less likely to notice and encourage mathematics talent in females. Schools tend to reinforce these social expectations. Fox also observes that media and textbooks, even math texts, tend to reinforce traditional sex-role stereotypes. She also notes the lack of career education programs and counseling services aimed at the special needs of women students.

What did the three papers recommend for new research?

In her recommendations for further research, Sherman stresses the need for broad-based research on the cognitive development of females. Noting a lack of studies using female subjects, she urges investigators to include both sexes and to report findings by sex. Fennema recommends interdisciplinary research teams of scholars and practitioners, longitudinal studies, and studies that are interrelated rather than independent. Funded studies should investigate the major variables related to females' decisions to study or not study mathematics, the scope of the problem nationwide with special populations of women, differences in types of mathematics courses taken by the sexes, and the differential reinforcement by teachers according to the sex of the student. In addition, some studies should analyze the relationships between verbal, visual, and mathematical skills.

Fox recommends that intervention programs aimed at increasing girls' participation in mathematics should be evaluated to identify successful components, such as female role models, non-sexist counseling, career education programs, grouping by ability or interest, and a variety of instructional modes. Interventions aimed at students should be compared to those aimed at parents or teachers. She also recommends that funded studies use uniform pre- and post-test measures and stresses the need for longitudinal studies. Also needed are studies on the development of career interests and aspirations, the development of attitudes characterizing mathematics as a male domain, and the development of math anxiety and low confidence in mathematics ability.

It is evident from these recommendation that much research is needed to understand the complex nature of influences affecting girls' participation and achievement in mathematics and to identify those strategies that work best at expanding the range of educational and occupational options considered by women.

What are the future activities of the women and mathematics program?

The women and mathematics program sponsored a grants competition for two-year research studies beginning in fall, 1977. These studies will investigate the major factors

affecting women's participation and achievement in mathematics in grades 7 through 12 and will identify how education can increase women's perceived occupational and educational choices. During the two years of funding, NIE will facilitate cooperative efforts and exchange of information among the funded researchers and others working in this area. The two-year grants program will be followed by a year of analysis, synthesis and dissemination during which researchers and practitioners will develop guidelines for building effective intervention strategies for increasing women's awareness of the importance of mathematics for future educational and occupational choices.

Judith Sauls Shoemaker, Ph.D.
Program Officer
Education and Work Group
National Institute of Education

BIOGRAPHICAL NOTES

**DR. LYNN FOX** is currently Project Coordinator of the Intellectually Gifted Child Study Group at Johns Hopkins University, a project sponsored by a grant from the Spencer Foundation, and Assistant Professor of Education, Evening College and Summer Session, Johns Hopkins. After receiving a bachelor's degree in mathematics education from the University of Florida in 1965, she taught mathematics to mathematically talented students and was an Education Specialist with the U. S. Office of Education. She continued her education in psychology at Johns Hopkins University, earning her doctorate in 1974. Her doctoral dissertation was titled "Facilitating the Development of Mathematical Talent in Young Women." Her publications indicate her continuing interest in intellectual giftedness and the development of mathematical talent. She has contributed chapters in several books, including Elizabeth Fennema's _Mathematics Learning: What Research Says About Sex Differences_. She was co-editor, with J. D. Stanley and D. P. Keating, of and a major contributor to _Mathematical Talent: Discovery, Description and Development_. Dr. Fox has written for the _Journal of Special Education_ and _School Science and Mathematics_ and has given numerous presentations at meetings of the Council for Exceptional Children, National Association for Gifted Children, American Educational Research Association, American Psychological Association and American Association for the Advancement of Science.

**DR. ELIZABETH FENNEMA** received a doctorate in curriculum and instruction in mathematics education from the University of Wisconsin-Madison in 1969. She has taught elementary school, served as a mathematics consultant to several school districts, and received two National Science Foundation grants from 1974 to 1976 to investigate the relationships between mathematics learning, sex, spatial visualization and social-cultural factors. She is currently Associate Professor, Department of Curriculum of Instruction, Women Studies Program at the University of Wisconsin-Madison. She has written extensively on mathematics in _The Arithmetic Teacher, American Educational Research Journal_, and _Journal for Research in Mathematics Education_. In 1975 Dr. Fennema edited _Mathematics Learning: What Research Says About Sex Differences_. She is active in several professional organizations, including the National Council of Teachers of Mathematics and the American Educational Research Association and most recently served as chairperson for AERA's special interest group on Research on Women in Education.

DR. JULIA SHERMAN is a psychologist who specializes in the psychology of women. Now director of the Women's Research Institute of Wisconsin and lecturer at the University of Wisconsin-Madison, her varied career has included work as a counselor, psychologist, lecturer and consultant to the Women's Professional Ser ice Corps, War on Poverty. She has served on a task force on Sex-Bias and Sex-Role Stereotyping in Psychotherapeutic Practice, sponsored by the American Psychological Association; and in 1974-75 she investigated intellectual and socio-cultural factors influencing election of mathematics courses and mathematics achievement among high school and middle school male and female students under a National Science Foundation grant. She wrote On the Psychology of Women: A Survey of Empirical Studies, co-edited with F. Denmark Psychology of Women: Future Directions of Research, and served as associate editor of Psychology of Women Quarterly. She has published numerous articles in journals such as Human Sexuality, Perceptual and Motor Skills, Psychological Review, The Arithmetic Teacher and American Educational Research Journal. Her education includes a doctorate in clinical psychology from the State University of Iowa in 1957.

# THE EFFECTS OF SEX ROLE SOCIALIZATION ON MATHEMATICS PARTICIPATION AND ACHIEVEMENT

By

Lynn H. Fox*
Assistant Professor of Education
and Project Associates of the
Intellectually-Gifted Child Study Group
Evening College and Summer Session
The Johns Hopkins University
Baltimore, Maryland 21218

Although males and females do not appear to differ on measures of general intelligence, sex differences on tests of mathematical achievement are repeatedly found in adolescent and adult populations (Aiken, 1976; Anastasi, 1958; Astin, 1974a; Fennema, 1974a; Fox, 1975a; 1975b; Maccoby and Jacklin, 1974). Dwyer (1976) has suggested that the tests such as the Scholastic Aptitude Test-Mathematics (SAT-M) may be biased against females due to artifacts of test-item construction and selection. Also, the question of differential aptitude is difficult to research because of the limitations of measurement tools and the overlap among measures of aptitude and achievement.

Fennema and Sherman (1976) suggest that there may be no real differences in aptitude at all and that sex differences on measures of aptitude and achievement reported in much of the literature may be the result of a failure to control for differential course-taking. Large sex differences have been found in favor of males, however, among gifted students on the Scholastic Aptitude Test-Mathematics (SAT-M) as early as grade seven (Astin, 1974a; Fox, 1975a). These sex differences can not be explained in terms of differential course-taking. It is, however, possible that differential exposure to mathematical games and activities outside of school accounts for some of the performance differences (Fox, 1975a; 1975b).

* I wish to thank Project Associates Linda Brody and Dianne Tobin, for their help in researching materials and editing the final draft of this paper. My thanks also go to Ilse Harrop and Amelia Benson who showed great patience and expert typing, correcting and editing of the various drafts and the finished version of this paper.

The fact that far fewer women than men pursue careers in mathematical and scientific areas and achieve lower scores on tests of aptitude and achievement in these areas has, until recently, been accepted as a natural consequence of innate sex differences in aptitude for those fields. Stafford (1972) and Page (1976) have suggested a sex-linked hereditary hypothesis. A rival hypothesis of merit is that sex differences in mathematical aptitude in adolescents and adults are a result of social factors (Aiken, 1976; Astin, 1974a; Fox, 1975b; 1976a; 1976b).

Whether or not one believes that there are sex differences in aptitude, one must accept the fact that there are great sex differences at the higher levels of achievement in mathematics that cannot be explained in terms of differential aptitude alone (Anastasi, 1974). Many females who do appear to have the aptitude do not take the advanced courses (Ernest, 1976; Fox, 1974a; 1974b; 1976c; Haven, 1972; Sells, 1976). Farley (1968) has suggested that perhaps there would be no sex differences in mathematics achievement if course-taking was not optional in the high school years. Sells (1976) says that avoidance of high school mathematics courses, not ability, is the critical filter that keeps women out of many career areas including, but not only, those in pure or applied science and mathematics.

The primary focus on this paper is on understanding factors related to sex differences in mathematics achievement with particular attention to course-taking. Such a focus excludes an in-depth treatment of the process of general socialization and child-rearing practices. Some studies purporting to assess socialization influences on aptitude rather than achievement and course-taking have been included. By and large, the research has ignored social class and racial or ethnic variables. The interpretation and integration of the research literature has also been limited by problems of definitions of terms and different assumptions about the process of socialization and the meaning of sex-role identity.

The perception of the usefulness of mathematics for future educational and career plans and the support or lack of support from significant others appear to be the major factors associated with women's decisions to elect or not elect advanced courses in mathematics. These factors are in turn influenced by the stereotype of mathematics as a male domain. Other factors associated with course-taking and achievement are attitudes towards mathematics, feelings of self-confidence, and values. Certain educational policies and practices tend to reinforce sex-role stereotypes while some practices may promote greater course-taking and achievement.

Thus, the organization of the research reported in the body of this paper is as follows:

1. Perceptions of the Career Relevance of Mathematics
2. The Influences of Significant Others
3. The Perception of Mathematics as a Male Domain
4. Attitudes, Self-Confidence and Values
5. Educational Policies and Practices

Some redundancy of topics from section to section appeared unavoidable. Omissions of some relevant research has also, undoubtedly, occurred. The research reported also varies in quality and some important questions remain unanswered. For the most part, however, the research studies were rather remarkably consistent in support of the premise that sex differences in mathematics achievement results, at least in part, from social influence.

## 1. Perceptions of the Career Relevance of Mathematics

One reason why many females do not achieve their full potential in mathematics is that they simply fail to elect to take the advanced courses in the secondary school. For some this decision may occur as early as grade seven when Algebra I becomes an optional course for the eighth grade. In many school systems, two years of high school mathematics are all that is required. Even females who elect a college preparatory program in high school are not typically required to take Calculus. Several studies indicate that females, even those who are college bound, elect not to take the advanced mathematics courses because they do not perceive them as useful to their future educational and career plans.

### Course-taking and Achievement

Haven (1972) found that the two most significant predictors of taking courses in mathematics in high school for above-average-ability girls were the perception of the usefulness of mathematics for future educational and career goals, and greater interest in natural sciences than the social sciences. Sherman and Fennema (1976) also found that course-taking in high school was related to the perception of the usefulness of mathematics. In a study of women mathematicians, Luchins (1976) found early career commitment among a third of the group. Almost all of the mathematicians had felt a strong interest in mathematics before entering college and a third, before age 12.

Fennema and Sherman (1976) found significant sex differences on achievement tests in two high schools where there were also significant sex differences in the perceived usefulness of mathematics, but no such differences in achievement in the two high schools where there were no differences in the perceived usefulness of mathematics. Hilton and Berglund (1971) also found significant sex differences in the perceived usefulness of mathematics in grades nine and eleven. They also reported a significant relationship between the increase in sex differences on achievement tests and perceptions of the usefulness of mathematics for earning a living. Fox (1975c) found significant sex differences in the perceived usefulness of mathematics in a study of gifted seventh and eighth graders. There were also significant sex differences on measures of aptitude and career interests in the population. Although the perceived usefulness of mathematics was not significantly related to the expressed willingness to accelerate in mathematics for the sample studied, it may be related to their future willingness to take advanced courses. Hilton and Berglund (1971) concluded that sex differences in the perception of the usefulness of mathematics resulted from the sex-typing of mathematics and careers.

The perceived usefulness of mathematics appears to be different for boys and girls for three related reasons. First, girls are less oriented to careers other than homemaker than boys. Second, girls who express career interests are more likely to be interested in fields other than mathematics and science than boys. Third, girls who are interested in careers in business, nursing, education and the social sciences, etc., are likely to be unaware of the relevance of mathematics and science to these professions. Sex differences in mathematics achievement are rarely found in elementary school populations. If courses were optional in those years, perhaps sex differences would be found sooner.

## Career Orientation

Sex differences in career interests are found as early as kindergarten and first grade (Looft, 1971; Schlossberg and Goodman, 1972). In one study, girls and boys were asked to describe a typical day in their life as an adult. Even the girls who had initially indicated a career goal other than homemaking described their typical day as being spent as a wife or mother, rather than at a job outside the home (Iglitzen, 1972, 1973). In a 1976 study of national merit scholars, sex was the single best predictor of career aspirations (Ory and Helfrich, 1976).

In an article entitled "Women in Science: Why so Few", Rossi (1965a) speculated that it is the belief that marriage and careers are not really compatible for women that lies behind girls' vacillation between the pursuit of social popularity (the perceived road to a successful marriage) and excellence in scholarship (the perceived road to a successful career). Astin (1974b) concluded that if a woman anticipates a conflict between the homemaking role and a career, she is likely to forego the career. Ory and Helfrich (1976)

found that females who held non-traditional sex-role stereotypes were more likely to desire professional careers than those who held traditional views of sex-roles.

Smith (1976) asked girls what they perceived as barriers to career plans; over half the respondents listed the following barriers: first, a long term commitment to a career interferes with raising a family; second, women have less geographic and thus career mobility because they must adjust their careers so as not to interfere with their husbands' success; and third, a strong career commitment would interfere with a happy marriage.

It is interesting to note that the gifted men in Terman's longitudinal study of genius who were married actually reported their greatest life satisfaction was derived from their family, followed by their career. The majority of these men were professionals, many in science areas (Oden, 1968 ; Sears, 1976). Perhaps we need to make this finding known and see if it is replicated in other samples. Clearly, the fact that these men gained their greatest satisfaction from the family did not inhibit their success in their careers (Sears, 1976; Oden, 1968). A follow-up study of the gifted women in Terman's sample found that the women who had careers derived great satisfaction from them, whether or not they had families (Sears and Barbe, 1975). Yet, the prevailing myth is that, while women may work outside the home, a full-fledged career will detract from their family life and, thus, their total fulfillment.

Hawley (1971; 1972) and Astin (1974b) concluded that marriage was the primary goal of most women, and if they do elect a career, it is based on what they feel men can tolerate. Astin (1974b) finds that while husbands of career women may be tolerant or even supportive of their wives, professional men in general hold negative attitudes towards women who try to fulfill the dual role of career woman and homemaker. Entwisle and Greenberger (1972) concluded that peers, especially male peers, exert considerable pressure on adolescent girls' occupational aspirations. White middle-class adolescent males were the most conservative in this respect. Although black adolescents were more liberal than whites, they are not strongly in favor of leadership positions for women. It is interesting that so many males perceive a conflict between family and career responsibilities for women, but not men.

Astin (1976b) concluded that there was little research evidence on how and why differences exist between women who decide to work and those who do not. One noteworthy difference that has been found repeatedly is that girls who persist in career interests from high school to adulthood are likely to score higher on mathematical aptitude tests than less career oriented girls (Astin, 1968; Astin, 1974a; Astin and Myint, 1971). This

suggests that autonomy and independence are associated with both career interest and mathematical competence. (Additional support for this hypothesis is discussed in the sections on parental factors.)

Although Astin (1974b) concluded that there was a paucity of good research on sex role identification and early socialization as related to career choice, there are a few interesting relationships and trends suggested by the literature. Presumably, women pattern their career expectations after the women they see. Girls see their mothers staying home and their fathers going to work (Rossi, 1965b). Lack of appropriate role models for girls limits their career choices (Lipman-Blumen, 1975). Astin, Harway and McNamara (1976) speculated that many young girls may be thinking about a career, but they have no experiences to help them know what is involved in the role of the career woman, so they focus instead on the female roles of wife and mother with which they are familiar. Women who go on to graduate school are likely to have had working mothers. Astin (1969; 1974b) concluded that either identification with the father, or a working mother are factors that influence a girl's career orientation. Girls who perceive their primary future role as wife and mother are certainly not likely to see advanced mathematics courses as relevant to this goal.

Career Interest in Mathematics and Science

If role conflict and absence of models are major barriers to development of career interests in general, they are even greater barriers to the development of career interests in mathematics or scientific fields. Professional careers in the sciences and mathematics are perceived as too demanding for women who wish to combine a family and a career. Many scientific careers require long years of training and do not lend themselves to interruption during the child bearing years (Rossi, 1965a). Rossi (1965b) found that female college graduates believe that women do not select careers in medicine, science, and engineering because they believe that these fields do not offer part-time or intermittent work. Astin (1969) found that college women in general do not pursue graduate training. Thus, they avoid careers in law, medicine, and science that require advanced degrees. Prediger, McClure and Noeth (1976) found that ninth and twelfth grade girls expressed doubts about the feasibility of combining family life with a career in science. Although they lacked information about specific steps in preparing for a science career, they did believe that the preparation would be long and difficult. Even mathematically gifted seventh and eighth graders differ drastically from boys with respect to interest in careers in mathematics and science (Fox, 1974b; 1975d; Fox and Denham, 1974). Fox, Pasternak, and Peiser (1976), however, did find that gifted seventh grade girls are more likely to be interested in science and mathematics careers than are average ability female adolescents.

The absence of role models and the presence of negative stereotypes appear to contribute to the lack of interest in scientific careers. Luchins (1976) reports that the stereotype of the mathematician is a person who can only communicate with other mathematicians. Unfortunately, this stereotype is reinforced by the isolation of research mathematicians. They are not highly visible to the general public. A study by Mead and Metraux (1957) found that the composite image of the scientist held by adolescents was of a person who was intelligent and dedicated, but whose work was dull and who neglected family and other interests. Ahlgren and Walberg (1973) also found the stereotype of scientists bo be remote and unsocial for high school students in physics classes. These negative stereotypes are hardly inspiring to adolescent girls who believe that marriage and family life are important.

Plost and Rosen (1974) found that eighth grade students exposed to a slide-tape presentation depicting men and women in two previously unfamiliar occupations in the computer science field preferred the one depicted by the like-sex model. The effect of like-sex model was significantly more potent for girls than boys. Males and females alike tended, however, to assign more prestigious characteristics to the occupation depicted by the male. This seems to raise the question, not analyzed by the authors, whether the girls prefer the occupation because they identified with the female role model, or because of the lower status they assigned that occupation by virtue of the female model. The students were simply asked to express a preference for one of the two occupations depicted, both of which were within the general computer area. Whether or not the students were seriously interested in the careers depicted is not known.

Lantz, West and Elliott (1976) reviewed projects funded by the National Science Foundation designed to increase the participation of girls in mathematics and science, and concluded that role models appeared to be the most effective component of some of the projects. In a project designed to recruit women to engineering careers, Leonard, Fein, Freim, and Fein (1976) found that the female role models who lived with the freshwomen in their dormitories may have helped reduce the feeling of discrimination towards females in engineering that the students felt at the beginning of the program. Thompson (1976) felt that seeing real women in these kinds of jobs made the girls more positive towards majoring in scientific areas. Levine (1976) interviewed female mathematicians and cited several instances where the women mentioned female role models as being important in their decision to continue studying mathematics. Smith (1976) reported that role models helped reduce the perceived conflict between parenthood and careers. Casserly (1975) found that girls who had an apprenticeship in a museum with a doctoral student became extremely enthusiastic about science. They reported:

"We were junior partners in her quest." Luchins (1976) reported that a woman mathematician felt that encouragement from a female mathematics professor had meant a great deal to her. Luchins suggested that women are not aware of female mathematicians who have "made it", and that books on the history of mathematics should pay more attention to female mathematicians.

It is not possible to say how lasting the influence of a short-term contract with one or more salient role-models will be. Career education models which do not employ live role-models appear to be less effective than those that do. Prediger, McClure, and Noeth (1976) sent information to twelfth grade women about programs in science and technology at colleges chosen by the students. A booklet depicting women in interesting careers in science and technology was also enclosed. This program was not found to be successful in increasing the students' interest in such careers or majors. Prediger et al. (1976) also experimented with a non-sexist interest inventory supplemented with group discussions of career planning in general, and science and technology in particular, with ninth-grade girls. This form of intervention was also not successful in changing career interests.

There is a clear need for more research on career education program models for women. Personal communications from Prediger stated that the failure of the two career intervention attempts may have been in part because ninth and twelfth grade is too late for effective intervention. The precise age at which intervention is necessary is difficult to assess. Even kindergarten children have stereotyped perceptions of women's work (Schlossberg and Goodman, 1972). These views are continuously reinforced throughout the school years. Perhaps career awareness programs should begin in the early elementary school years, followed by more intensive programs in the middle and high school years.

Knowledge About Careers

Fennema and Sherman (1976) have suggested that girls are unaware of the relevance of mathematics to many career areas other than purely scientific ones. Anecdotal accounts support this view. More systematic study of this question seems warranted. In a pilot study of career awareness and mathematical skills, fifth and sixth grade boys and girls seemed to be relatively naive about the uses of mathematics. After exposure to a program that emphasized the applications of mathematics to art and social problems the girls became considerably more positive about school, reported liking mathematics more, and showed some increased interest in scientific careers (Fox, 1976d). A somewhat different but related mathematics program is now being tested for college students at Wellesley (Schafer, 1976). The results of this approach will be interesting.

Knowledge about the applications of mathematics to real life is sorely lacking in most mathematics curricula. Work problems in algebra, for example, that deal with rowing boats up and down streams or draining vats with different sized pipes are not very relevant to many life situations. The introduction of courses in computer science and applied statistics at earlier ages might make mathematics seem somewhat more meaningful.

When girls are faced with decisions about taking advanced mathematics courses, they are likely to seek guidance from significant others in their lives, such as teachers, guidance counselors, peers, and parents. Girls are also likely to be influenced by indirect, as well as direct, messages they received from these important others in their lives and the society as a whole about the relevance of careers in general, and achievement in mathematics in particular.

## 2. The Influence of Significant Others

The impact of significant others upon course-taking and achievement in mathematics and career aspirations seems worthy of special attention. Parents, peers, teachers and counselors have tremendous influence upon the child and adolescent learner. Unfortunately, very little research on child-rearing practices has focused specifically on the development of competence in mathematics. In this section, some general discussion of the child development literature will be interwoven with some studies of the specific impact of socializing agents upon mathematics achievement. The presentation of the data on the impact of significant others is given in reverse order of apparent potential for influence. Thus, counselors are perceived as the least potent influence, and parents as the most significant others.

### Counselors

Harway, Astin, Suhr, and Whiteley (1976) reported that females were more likely than males to seek advice from counselors, more than any other source, regarding advanced mathematics courses. Numerous sources suggest that the girls receive little encouragement from counselors to pursue mathematics courses or "deviant" career goals (Casserly, 1975; Harway, et. al., 1976; Christman, Vidulich, Dralle, and Kirk, 1976; Haven, 1972; Luchins, 1976, Perucci, 1970; Scholossberg and Pietrofisa, 1974; Friedersdorf, 1970). For example, Haven (1972) found that 42 percent of the girls who were interested in careers in mathematics or science reported being discouraged rather than encouraged to take advanced mathematics courses. Over half of the girls who did not take the courses reported no encouragement to do so from counselors. Whether or not they were actually discouraged is not clear. Casserly (1975) and Luchins (1976) reported that counselors actually admitted discouraging girls from taking the advanced courses. The reasons counselors gave reflected their stereotypes. For

example, counselors said that they discouraged girls from taking the courses because a low grade would hurt the girls' otherwise excellent school records. Other reasons were that jobs in the sciences were scarce and should go to men, or that such careers were too demanding for women. Perucci (1970) noted that even in some elite women's colleges, counselors discourage women from careers such as engineering and medicine.

Casserly (1975) suggested that female counselors may be projecting thier own anxieties about careers in mathematics and science onto the women they counsel. Harway, et al. (1976) suggested that counselors are either ambivalent or negative. Thomas and Stewart (1971) said that girls who are contemplating "deviant" careers may find themselves confused by the stereotypic thinking of their counselors.

One bright spot is that students may not be strongly influenced by their counselors. Harway et al. (1976) reported that the majority of students (80 percent) felt that counselors exert little or no influence. Casserly (1975) reported that in only five of thirteen high schools, students reported seeing counselors as a positive force in their lives. Haven (1972) also found that many girls ignored their counselors' advice about mathematics courses. It is also important to note that stereotyped thinking may be declining among counselors. Englehard, Jones, and Stiggins (1976) reported that counselors' attitudes in 1974 were less stereotyped than in 1968. Changes in counselor attitudes should not be left to chance, however. Perhaps "counseling" for counselors is needed.

<u>Teachers</u>

There is some controversy over the effect of teacher expectation upon student achievement as suggested by Rosenthal and Jacobson (1968). In some studies, large effects have been found (Dusek, 1975; Lockhead, 1975). The support or lack of support of teachers does appear to have an effect on female achievement in mathematics. A very negative attitude towards mathematics is often linked to bad experiences with a teacher (Ernest, 1976; Haven, 1972; Poffenberger and Norton, 1956, 1959). Inspirational teachers, on the other hand, were frequently cited by women mathematicians as a major factor in their choice of career (Luchins, 1976). Studies by Casserly (1975) and Anderson (1963) also showed the positive effects of non-sexist and enthusiastic teachers upon the intellectual development of girls.

Unfortunately, teachers appear to have different expectations for girls and boys in mathematics at the high school level (Ernest, 1976; Levine, 1976). Solano (1976) found that teachers have more negative perceptions of mathematically gifted girls than boys.

Fox (1974b, 1976c) found that teachers can be hostile towards mathematically gifted girls. Frazier and Sedaker (1973) have referred to teachers as "hidden carriers" of society's sex-role stereotyping. Such stereotypic thinking appears to be common among both female and male teachers.

Studies of student - teacher interaction indicate that teachers interact more with males than females, especially in mathematics and science classes (Bean, 1976; Good, Sikes, and Brophy, 1973; Levy, 1972; Sanderson and Anderson, 1960; Stacey, Bereaud and Daniels, 1974). Female students appear to be more sensitive to corrections by teachers (Dweck, 1976; White and Aaron, 1967). It seems likely that most teachers are "unconscious sexists" and should be made aware of the negative outcomes of their sex-role stereotyped attitudes and behaviors. When teachers actively recruit girls for mathematics programs and expect them to perform as well as males, the results are significantly positive (Casserly, 1975).

The impact of teachers upon students seems to be most potent at the extremes of attitudes. Thus, a very bad experience with a teacher may be influential; and when a teacher makes a very strong effort to support and encourage, it may also be influential. The overall impact of teachers may be less potent than the influences of others. Additional research on the influence of teachers will be presented in the section on attitudes.

## Peers

Maccoby and Jacklin (1974) asserted that boys and girls were equally affected by social stimuli. They, like Coleman (1961), however, noted that girls gravitate towards small same-sex cliques while boys function as loners or in larger, more flexible peer groups. The pressure to conform is somewhat greater in the smaller peer group. Shapiro (1962) found that upper elementary grade girls (grades four to six) were more susceptible to peer influences than were boys.

In the elementary school years, Ernest (1976) found that students believe their own-sex peers are superior in all subjects. In early adolescence, gifted girls and boys tend to see both sexes as equally talented in mathematics (Fox, 1975c). In the high school years, however, males and females are more likely to perceive mathematics as a male domain (Ernest, 1976; Fennema and Sherman, 1976; Sherman and Fennema, 1976).

Thus, in adolescence, girls may perceive real peer pressures against achievement in mathematics. Solano (1976) found that adolescents have a more negative stereotype of mathematically gifted girls than boys. In adolescence and young adulthood, the peer pressure against achievement in mathematics may be directed more strongly from male than female peers, at least in some situations. Entwisle and Greenberger (1972) found middle-class males

were less supportive of achievement for women than were the girls themselves or males from the working class. Fennema and Sherman (1976) found males more than females stereotyped mathematics as a male domain. Husbands (1972) found that female college students were more concerned about male than female perceptions of their femininity. Girls did not socialize or date the boys in their advanced placement classes in the Casserly (1975) study.

Anecdotal accounts illustrate how peer pressures can operate. One mathematically gifted girl dropped out of an accelerated mathematics program only because her best friend did so (Angel, 1975). Although many mathematically gifted males skip grades or take college courses early with little or no problem (Stanley, 1973), mathematically gifted girls are very reluctant to skip a grade or take college courses early because of fear of peer rejection. One girl was ready to abandon a grade-skip in the first week of school and returned to the lower grade because she had no friend with whom to eat her lunch (Angel, 1975). Casserly (1975) found that girls who took advanced placement courses in mathematics remarked on the importance of a girlfriend's support to help deal with the disapproval of boys.

Achievement and course-taking in mathematics and science appear to be influenced by the sex-ratio of the learning situation. Efforts to recruit girls for special mixed-sex mathematics classes outside of school were not very successful when the number of girls was small. When an all-girl class was arranged, the response was considerably higher (Fox, 1974a; 1974b; 1976c). Ruehl (1975) found that girls paired with girls in a physics lab were more successful than boys paired with boys, or mixed-sex pairings. Presumably, the all-girl classes or pairings allow girls to achieve by reducing sex-role conflict. Hurley (1964); 1965) found greater gains on achievement tests for girls in all girl classes at the fifth grade than for girls in mixed sex classes at the end of one year. This included a significant gain in arithmetic concept scores. Boys made few gains in the first year, but did better the second year. Teachers noted a significant decline in sex-role stereotyped behavior for both sexes in the same-sex classes. Ironically, the boys, but not the girls, wished to continue the same-sex classes. Why the girls did not like the sex-segregation is not known. Husbands (1972) argues against sex-segregation, at least at the college level, because the quality of the mathematics and science offerings will become diluted in all-girl classes.

Although all-girl classes may not be totally acceptable to girls, there may be some value in having programs that track girls early for academic programs that lead to the taking of advanced courses in mathematics and science (Fox, 1975b; 1976a). Both Haven

(1972) and Casserly (1975) found this to be true. Being "tracked" as early as grade four may instill in the student the expectation that an academic program, including advanced mathematics and science, is inevitable. Caserly (1975) found that for girls not tracked early, the placement in Algebra in the eighth grade was often traumatic. Early tracking also may lead to the development of a peer clique of girls who support and reinforce each other. Farley (1968) went even further and recommended that four years of mathematics be required for everyone. This would eliminate course options and diffuse the mystique of mathematics as a male domain. If choice is eliminated, girls would not have to defend their pursuit of such courses to male or female peers.

Attitudes of peers may be more important in reinforcing existing stereotypes and attitudes than changing attitudes and behaviors (Almquist and Angrist, 1971). Poffenberger and Norton (1956; 1959) noted that basic attitudes and beliefs about mathematics are shaped before the child enters school. It is generally believed that the prime agency of socialization is the family.

## Parents

In a review of the general literature on socialization and sex-typing Maccoby and Jacklin (1974) concluded that there is "a remarkable degree of uniformity in the socialization of the two sexes". The major sex difference is that boys seem to have a more intense socialization experience than girls. They are more pressured than girls against engaging in sex-inappropriate behavior. These conclusions are based on sex-typed behavior in narrowly defined terms, such as toy preference and clothing. Parents reportedly notice children's behaviors more when they run counter to sex-role stereotypes and they punish wrong behavior more when it is seen as sex-inappropriate than sex-appropriate, while reinforcing desirable behavior more when it is seen as sex-inappropriate rather than sex-appropriate. Maccoby and Jacklin (1974) refer to this as the "perceptual adaptation level" hypothesis. They cited no studies, however, relating this to mathematics achievement.

There is some evidence that support and encouragement from parents are crucial for girls in their decisions to elect or not elect mathematics courses in high school (Haven, 1972; Luchins, 1976; Poffenberger and Norton, 1956; 1959). Fox (1975c) found that mathematically gifted boys were significantly more likely to perceive their parents as favorable towards acceleration in mathematics than were girls. Anecdotal evidence from the Study of Mathematically Precocious Youth indicates that the girls are correct in perceiving less parental support (Stanley, Keating and Fox, 1974; Keating, 1976). Fennema and Sherman (1976) also found sex differences in students' reports of parental perceptions of the child as a learner of mathematics.

The influence of fathers may be even greater than that of mothers. Haven (1972) found that girls were more likely to discuss course-taking with fathers than mothers. Ernest (1976) found that at the secondary school level, both males and females turned to the fathers rather than the mothers for help with mathematics homework. Carlsmith (1964) found that mathematics achievement was lower for both boys and girls in father-absent homes than in homes where fathers were present. Helson (1971) suggests that fathers were more salient parental figures than mothers in homes of woman mathematicians. Block (1973) concluded that fathers were more potent agents of sex-typing than mothers.

In general, parents are less likely to notice and encourage mathematical talent in female offspring. Astin (1974a) found that parents of mathematically gifted boys were more likely than parents of girls to notice their offsprings' talent in early childhood. Also parents of boys bought toys of a scientific nature for their sons more often than did parents of girls. The provision by parents of more toys, games, etc. of a scientific nature for boys than girls was also reported by Maccoby and Jacklin (1974). Casserly (1975) noted that girls in her study felt that they had received fewer such toys than their brothers. Career goals and educational plans were more likely to be discussed with children by the parents of boys than by the parents of girls.

Levine (1976) found that parents hold lower educational aspirations for daughters than for sons. Casserly (1975) also found that parents had limited aspirations for daughters. Levine (1976) found that 75 percent of the mothers of girls who earned poor grades in mathematics accepted this as inevitable because of their own lack of ability in mathematics. Parents of boys who earned poor grades in mathematics were likely to say that the boy was simply lazy. Thus, lower levels of achievement in mathematics are more easily accepted by parents of girls than parents of boys. Helson (1971) found that female mathematicians in the United States, particularly creative ones, were likely to come from all-girl families and/or were first or second generation Americans. This suggests that parental expectations for daughters are influenced by social factors. If family resources for education are limited, the presence or absence of a male child may influence the degree of support and encouragement a female child receives. Casserly (1975) found that even in working class families where the aspirations for daughters were high, the understanding of how to achieve the goal was missing.

Several studies suggest that the development of a low numerical ability pattern or its converse result from specific child-rearing practices. Bing (1963) found that discrepant verbal ability was fostered by a close relationship with a demanding and intrusive mother, while discrepant nonverbal abilities are enhanced by allowing a child a considerable degree of freedom to experiment on his or her own. A marked pattern of help seeking and help giving interferes with the development of an independent, self-reliant attitude.

This, in turn, is needed for non-verbal competency. Unfortunately, Bing did not control for hereditary influences. Perhaps non-interfering mothers are themselves less verbal. Other studies have, however, found similar results. Ferguson and Maccoby (1966) found that high number ability boys and girls were detached from their parents from over-close dependency. High verbal/low number ability boys and girls were overly dependent upon their parents. Block, (1973) reported that parents of boys emphasize achievement, competition, and control of feelings and expression of affection. Parents of girls emphasize interpersonal relationships, affection and protection. Although White (1973) has suggested that there is a generalized parenting style that fosters competence in children, Baumrind (1972) notes that a parental style may have differential outcomes for boys and girls. Stein and Bailey (1973) concluded that moderate but not overpowering parental nurturance, permissiveness, and encouragement of independence and achievement efforts were aspects of child-rearing practices that facilitated the development of an achievement orientation in women. The presence of an achieving maternal model was also considered to be an influential factor.

Aiken (1975 ; 1976) hypothesized that sex differences in mathematical ability was a partial consequence of same-sex modeling. Young girls model their behavior after their mothers who are typically more verbal than quantitative. This, combined with the general lack of salient female models who exhibit mathematical orientations, leads girls to view themselves as incompetent in mathematics. Maccoby and Jacklin (1974) conclude that same-sex models are more potent for stimulating imitative behavior. The exact process by which boys and girls learn sex-role appropriate behaviors, however, is not clearly understood; it may not result from direct modeling but rather from a more complex cognitive process. (Kolberg, 1966; Maccoby, 1966; Maccoby and Jacklin, 1974; Mischel, 1966). The hypothesis that mathematical competence results from identification with a masculine role is discussed in the following section.

### 3. The Perception of Mathematics as a Male Domain

The belief that mathematics is a male domain reaches the very core of the issue of sex-role socialization and mathematics achievement. Two major hypotheses often offered as social-cultural explanations for sex differences in mathematics are the masculine identification hypothesis and the cultural reinforcement hypothesis (Aiken, 1970a; 1975; 1976; Astin, 1974a). In the former achievement and interest in mathematics are assumed to result from identification with the masculine role. This is related to the issue of achievement motivation. The second hypothesis also assumes that mathematics is a masculine domain, and thus females receive less encouragement from society for achievement in this realm. The support of significant others has already been discussed. In this section the emphasis is upon the relationships between the sex-typing of mathematics and the encouragement from significant others.

## The Masculine Identification Hypothesis

A major hypothesis offered as an explanation for sex-differences in mathematics achievement is the masculine identification hypothesis (Plank and Plank, 1954; Elton and Rose, 1967; Aiken, 1970a; 1975; 1976). It is argued that boys and girls who identify with their fathers or a generalized masculine sex-role are either better at mathematics than those who have a feminine identification, or at least are better mathematically than verbally. Alas, the studies which attempt to deal with this issue are not in total agreement. A major source of confusion lies with the definition of masculine identification. Most studies used a score on a paper-and-pencil test of masculinity/femininity as the measure of sex-role identification. Constantinople (1973) has stressed the basic problems with bipolar scales and constructs of masculinity and femininity. Persons who score relatively low on the sex-appropriate scale may be androgynous rather than cross-sex identified.

Block (1973) reports that women who score high on the masculinity scale of the California Psychological Inventory (CPI) can be further divided into two groups based on their socialization scores. Women who are high on socialization appear to have a general feminine identification but without the "female" passivity and dependency characteristics despite their "masculine" CPI scores. Analysis of the life history of these women suggests that they did not identify specifically with either parent, but with an "androgynous" composite of the parents. Those women exhibited extreme deviations from traditional definitions of appropriate sex role.

Heilbrun (1974) in a lengthy review of the literature on parent identification and filial sex-role behavior concludes that the biological and psychological sex of the parent with whom the child identifies is related to sex-role identification of boys, but not girls. Girls who have identified primarily with either a masculine father or a feminine mother can be equally feminine in their own identification. Both Block and Heilbrun concluded that measures of either a masculine or feminine identification for women are independent of sex of parent with whom the child identified. It is argued that what is even more important is that sex-role identification relates to healthy psychological adjustment for boys, but not girls (Heilbrun, 1974; Broverman, Broverman, Clarkson, Rosenkrantz, and Vogel, 1970).

Thus, for females, identification with the father does not necessarily lead to a masculine rather than feminine identification and, conversely, high scores on masculinity scales do not necessarily mean identification with the father. This complicates the interpretation of studies on masculine identification and mathematics achievement.

The masculine identification hypothesis appears to have originated from a report by Plank and Plank (1954) on case histories of women who were eminent in scientific and mathematical fields. Clinical evaluations of women mathematicians rated as creative by their peers revealed that many of these women had identified with their fathers rather than their mothers. They did not, however, score extremely high on the masculinity scale of the CPI (Helson, 1971). It is difficult to draw conclusions on such small select samples. Perhaps the identification with father for these women is related more to their pursuit of atypical careers than the development of their mathematical ability per se.

Another study often cited as support for the masculine identification hypothesis is one by Carlsmith (1964) who found that boys and girls whose fathers were absent from the home during the early childhood years had higher verbal aptitude than mathematical aptitude. Carlsmith assumes the high verbal, low mathematical pattern is a feminine style while the reverse pattern is masculine. Thus, the absence of the father lessened the availability of the masculine model for identification. Landy, Rosenberg and Sutton-Smith (1969) also found decreased quantitative scores in father absent homes for girls as well as boys. Landy et al. suggest that it is decreased opportunity for interaction with the father that accounts for the lower scores.

Ferguson and Maccoby (1966) offered a different interpretation of the causes of discrepant ability patterns. They hypothesized that stress and tension interfere differentially with functions underlying mathematical and verbal performance. Parental absence causes tension and thus a high verbal and low mathematical profile for both sexes.

In a test of the two hypotheses (Carlsmith's sex-typing and Ferguson and Maccoby's tension hypothesis) Nelson and Maccoby (1966) did not get clear results and support of either hypothesis. They concluded that perhaps the hypotheses combined had some explanatory merit; for example, boys who had a cross-sex identification were experiencing more tension and the identification with the mother combined with the tension produced the high verbal pattern. Lynn (1972) suggests that boys are more analytical than girls because the very process of identifying with the abstract, culturally defined masculine role requires a more analytical method of learning sex-role than does the learning of the female role through direct identification with the mother. Presumably, girls who identify with the masculine role are also developing the more analytical cognitive style.

Kolberg's (1966) model of sex-role identification, however, implies that both sexes employ cognitive processes in the act of developing their sex-role identifications. A study by Brown (1957)

suggests that girls know the masculine role and prefer it to the female role in the early grades. By grade 5, girls begin to switch to a preference for the feminine role. If girls know the masculine role, they must, like the boys, learn about it in an analytical way.

Thus, there is no overwhelming evidence that identification with the father is a necessary condition for the development of mathematical competence. When masculine identification is defined in terms of psychological test scores, the results are also somewhat contradictory with respect to mathematical achievement.

In a study by Elton and Rose (1967), women with high mathematical aptitude and average verbal aptitude scored higher on the masculinity scale of the Omnibus Personality Inventory (OPI) than did women with average mathematical aptitude and high verbal aptitude. Women who were either high or average on both abilities did not score high at the extremes of the scale. The Masculinity/Femininity dimension used in this study was a measure of a theoretical versus aesthetic orientation. Milton (1959) found males and females who had masculine orientations were better problem-solvers than males and females who had less masculine orientations.

Somewhat opposite results are suggested, however, by other studies. Lambert (1960) found that female mathematics majors in college scored higher on the femininity sclae of the Minnesota Multiphasic Personality Inventory (MMPI) than did female education majors. (The number of female mathematics majors, however, was very small.) No differences were found between male mathematics and education majors on this scale. A study of eleventh-grade girls found that the higher achievers in mathematics were significantly more accepting of the female sex-role stereotype than were lower achieving females (Jacobs, 1974).

Research on the masculine identification hypothesis is difficult to interpret due to the measurement problems involved. Bem (1974) and others (Constantinople, 1973; Spence, Helmreich and Stapp, 1975) have suggested that masculinity and femininity should be treated as two dimensions rather than a bipolar scale. It may well be that psychological androgyny rather than a strong masculine identification is associated with mathematical competence in women. Thus, females who do not sex-type mathematics as masculine are able to achieve in mathematics without a conflict with their acceptance of a general feminine sex-role, whereas females who sex-type mathematics as masculine, and have a feminine sex-role identification experience conflicts and achieve less. For example, Bem and Lenny (1976) found that cross-sex behavior is motivationally problematic for individuals who are strongly sex-typed. Discomfort was greatest when the subject had to perform a cross-sex activity in front of a member of the opposite sex. This was not true of androgynous subjects.

At present the evidence that identification with the father or

or a masculine identification as measured by tests is a necessary condition for achievement in mathematics is not overwhelmingly impressive. The hypothesis that sex-typing of mathematics as a male domain inhibits female achievement seems to be a more logically consistent argument.

Achievement Motivation and the Male Domain

The sex typing of mathematics as a male domain is related, indirectly at least, to the literature on achievement motivation and women. Several researchers have noted the potential conflict for girls between academic achievement and popularity (Coleman, 1961; Hawley, 1971; 1972; Komarovsky, 1946). Lavach and Lanier (1975) found that success in a domain labeled "feminine" was less threatening for females than success in a domain labeled "masculine". As long as mathematics is considered to be a masculine domain, achievement conflicts are likely to exist for girls.

The construct "fear of success" proposed by Horner (1972) has been sharply criticized (Zuckerman and Wheeler, 1973). While it is true that attempts to replicate Horner's original studies and to expand the concept have met with problems, the theory does seem to fit well with anecdotal data on women and achievement in mathematics. For example, gifted girls have often been reluctant to accelerate their progress in mathematics because of fear of negative social consequences, primarily peer rejection (Fox, 1974b; 1975b; 1976c). Fennema and Sherman (1976) reported that girls said they did not pursue the advanced mathematics courses because taking such courses might hamper their social relationships with boys and/or make them appear masculine. Levine (1976) also reported that girls believed that boys do not like "smart" girls, especially "math whizzes". Sherman and Fennema (1976), however, did not find a "fear of success" effect in their study when they attempted a paper-and-pencil assessment in relationship to mathematics.

While not all women fear success Horner (1972) said those who have high fear of success were the least likely to develop their intellectual potential, especially in situations requiring competition with men. Romer (1975) found similar results for girls in high school. If this is true, perhaps these women should feel less threatened in all female classes for mathematics than in mixed-sex groups. Fox (1974b; 1976c) found that it was easier to recruit seventh grade girls for all girl accelerated summer mathematics classes than for co-educational ones.

There appears to be a need to test the relationships of fear of success and failure motives more directly within real and observable, rather than imaginary settings. The rejection of advanced mathematics classes by girls may be a result of complex motives. Some may fear the consequences of success in a situation they consider to be a male domain. Others may fear failure partly because they are conditioned to believe they are less capable than the males in this domain.

Some may fear both success and failure. They will appear "stupid" if they fail or ask a silly question but appear "masculine" if they are too successful. Another possibility is that girls do not fear either success or failure but simply see no value in achieving in a domain labeled as masculine by society.

## Sex-typing of Mathematics

There is little doubt that the perception of mathematics as a male domain is common. As Casserly (1975) points out, traditionally the physical sciences and mathematics have been male provinces, and relatively few women have crossed the borders to seek eminence in these domains. Lambert (1960) hypothesizes that even at an early age, boys are expected to be interested in mathematics. Girls, on the other hand, though they may have equal ability, may be discouraged from learning by the prevailing idea that mathematics is a masculine field. Ernest (1976) cites a professor at Columbia University, who said, "Why didn't I study Mathematics at age 21? I felt it was not a 'feminine' thing to do. I'm afraid that it seems to me that this is a continuing problem for many young women." Another professor said, "Many people, on hearing the words 'female mathematician' conjure up an image of a six-foot, grey-haired, tweed-suit, oxford clad woman. This image, of course, doesn't attract the young woman who is continually being bombarded with messages, direct and indirect, to be beautiful, 'feminine', and to catch a man."

While investigating high school students' attitudes toward mathematics, Sherman and Fennema (1976) addressed the issue of mathematics as a male domain. They reported that the boys, more than the girls, rated mathematics as a male domain. Sherman and Fennema (1976) hypothesized, however, that these girls live in a community where the women's movement receives much publicity. When the females in this study were asked to respond to such an item as "Studying mathematics is just as appropriate for women as for men" they agreed. They behaved, however, in a more stereotypic way when it came to course selection. The boys, on the other hand, may have felt freer to express the view that mathematics is a male domain. Sherman and Fennema (1976) suggested that this masculine view is undoubtedly communicated to the girls. Thus, while the study did not document that girls more than boys believe mathematics is a male domain, it did find that the girls' actions contracicted their words. The stereotyping of mathematics as a male domain did relate to both female achievement and course-taking (Fennema and Sherman, 1976).

Sherman and Fennema also point to the fact that mathematics teachers tend to be male (69 percent in the city studied in 1974-75) and this contributes to the impression that mathematics is a male domain. There is also a tendency for the more advanced mathematics courses to be taught by males, lending additional support to the overall impression that mathematical thinking is a male province.

In a study by Farley (1974) students were asked to rank in order

six reasons for girls' lack of interest in mathematics oriented work. The second choice for girls was that men do not want girls in the mathematical occupations. Boys ranked this reason fourth. Thus, this study suggests that girls feel more strongly than boys that there is male prejudice against the girls' engaging in mathematics-related work. Neither group put much stress on the idea that it is not ladylike for girls to enter these occupations. Both groups ranked this reason fifth. Farley suggests that this is a departure from the popular opinion of previous decades. She cites the case of a high school senior in a girls' school, who had done outstanding work in mathematics and science. When the student sought information about the procedure for applying for admission to M.I.T., she was referred to the school's consulting psychologist, who was charged with the duty of redirecting her interests into more ladylike channels.

Thus, the Farley study and the Sherman and Fennema study suggest that the concept of mathematics as unfeminine is not as prevalent as it once was, and that girls on the verbal level attest to their right to enter any field. The evidence, however, shows that male prejudice against girls' entering mathematics either still exists (as the Fennema- Sherman study suggests) or, at least, the girls believe it exists (as the Farley study suggests). The end result is the same. If male prejudice is inhibiting to a girl, that girl will probably choose to study something other than mathematics when given the choice.

Hawley (1971; 1972) found a definite relationship between women's career choices and their perceptions of significant men's views of the faminine ideal. She found that women preparing for traditionally feminine careers (e.g., teaching) believe significant men in their lives dichotomize attitudes and behaviors into male-female categories. They thought men viewed behavior as appropriately male or female. Those preparing for nontraditional careers (e.g., mathematics/science majors) believe men do not see sex as a determinant of attitudes and behavior. The math/science women indicated that their men feel women should be free to compete with men in all areas, even those that have been traditionally considered male domains.

Casserly (1975) found that many guidance counselors still believe that careers in mathematics and mathematics courses are male domains. The fact that teachers perceive boys as better at mathematics than girls (Ernest, 1976) is also suggestive of their sex-typing of mathematics.

## Parental Socialization

Since teachers, counselors and peers tend to reinforce the belief in mathematics as a male domain, it seems likely that this belief has early roots. Yet there is little research on parental sex-typing and socialization practices as they affect attitudes and/or achievement in mathematics. In a review of the general literature on socialization and sex-typing Maccoby and Jacklin (1974) concluded that there is "a remarkable degree of uniformity in the socialization of the two sexes". The major sex difference is that boys seem to have a more intense socialization experience than girls. They are more pressured

than girls against engaging in sex-inappropriate behavior. These conclusions are based on sex-typed behavior in narrowly defined terms, such as toy-preference and clothing. Parents reportedly notice children's behaviors more when they run counter to sex-role stereotypes and they punish wrong behavior more when it is seen as sex-inappropriate than sex-appropriate, while reinforcing desirable behavior more when it is seen as sex-inappropriate rather than sex-appropriate. Maccoby and Jacklin (1974) refer to this as the "perceptual adaptation level" hypothesis. They cite no studies, however, relating this to mathematics achievement.

The evidence that parents sex-type mathematics comes largely from indirect rather than direct measures. For example, Ernest (1976) found that after grade six, both males and females tended to seek homework help in mathematics from fathers rather than mothers. Parents of mathematically gifted boys were more likely to report having bought scientific and mathematical games and toys for their sons than were the parents of girls (Astin, 1974a). Block (1973) found fathers to be more crucial than mothers as the agent for directing the sex-typing of the child. (Although parental ideas about mathematics per se were not investigated, Block found that parents of boys emphasize achievement, competition and control of feelings and expression of affection, while parents of girls emphasize interpersonal relationships, affection and protection.) This suggests that changing the sex-role stereotypes of fathers is even more important than changing those of mothers.

If parents do sex-type mathematics as a male domain, this should lead to differential expectations of behavior for sons versus daughters. There is some evidence to this effect. Fathers who regard mathematics as a more masculine than feminine pursuit had higher expectations for their sons in mathematics than did fathers who sex-typed mathematics less (Hill, 1967). A similar relationship was not found for mothers. The effect of sex-typing of parents on expectations for daughters was not studied. Perhaps mothers and fathers who sex-type mathematics have lower expectations for daughters than parents who do not sex-type this domain. In general, perceptions of parents' expectations of the child's mathematical ability is lower for girls than boys. This perception leads to differential self-conceptions and course-taking (Kaminski, Erickson, Ross, and Bradfield, 1976). The relationship of this to sex-typing by parents is assumed, but not systematically shown.

Parents who sex-type mathematics as masculine are probably more likely to notice and reward success in mathematics and less likely to punish failure for girls than boys. There appears to be no study which effectively supports or reflects this hypothesis. A study by Astin (1974a) found that parents of mathematically gifted boys were more likely than parents of mathematically gifted girls to notice and reinforce their child's early interest in mathematics. The degree to which the parents sex-typed mathematics is not known.

There is however, a general belief that parents are more tolerant of failure in mathematics for girls than boys. This is supported by anecdotal reports more than direct research (Levine, 1976). In a study of pre-schoolers, Block (1973) found parents to be much more concerned with task-oriented achievement for boys than for girls. Block (1973) concluded that parents expect far less achievement from girls than boys in general.

Although there is a large body of literature on sex-role socialization and modeling, none of the theories are adequately proven (Maccoby and Jacklin, 1974). It appears that children learn by employing both direct modeling of same- and opposite-sex models and by generalizing, from observational learning, the behavior of both sexes. Unfortunately, there is little research evidence to suggest how attitudes and competence or lack of competence in mathematics results from the socialization practices of parents. We can only speculate. For example, if Kolberg's (1966) theory of sex-role development is correct, the observation of the sex-typing of mathematics as a masculine domain by parents should lead to the internalization of different self-expectations for girls and boys. If more direct modeling theories are correct, the fact that more fathers than mothers are likely to be the models for mathematical activities, as suggested by Ernest (1976) and Aiken (1976) will also lead to sex differences. (As noted earlier, Lynn's (1972) speculations on the development of analytical competence as a result of modeling on abstract rather than concrete role models is not supported by evidence.) The problem is that regardless of the dynamics, the message most girls are likely to hear is that competence in mathematics is a masculine rather than feminine trait.

## Media and Textbooks

Whether or not the female child has access to mathematically competent female role models in the home, the general communication from the larger society is sex-biased. Television, children's literature, textbooks, and tests typically reinforce traditional sex-role stereotypes (Harway et. al., 1976), some of them conveying some distinct messages about women and mathematics.

In a study on role models presented to children in children's TV shows, Sternglanz and Serbin (1974) pointed out that the role models for children on TV depict women in a derogatory manner, rarely having jobs, and usually in romantic and/or family roles. Moreover, commercial TV shows aimed primarily at children show males as aggressive and constructive. Females are shown as being second-rate and punished when they engage in a high level of activity. TV shows carry a different message for girls than boys, and they postulate that girls learn from their TV watching that it is inappropriate to make plans and carry them out or to be aggressive. Girls will be punished if they abandon the sedate female style. The most successful

route for females on TV shows seemed to be magic. Even Sesame Street, one of the most popular television shows for little children, reflects sex role stereotyping. Bergman (1974) states that a little girl watching Sesame Street "was like taking lessons in invisibility." Live females are underrepresented and generally appear in stereotyped roles and there are few female puppet characters who aren't mommys or sisters. Cartoons used in the program are narrated by males and almost all were about males. Documentaries rarely show girls doing anything except standard stereotyped play activities. Although Bergman believes the program has changed somewhat from being incredibly sexist to being slightly sexist, she feels it has not yet gone beyond tokenism.

Carney (1974) reports that racism and sexism are still the rule in children's books and textbooks. A National Organization for Women (N.O.W.) group read 135 books and children's readers in an effort to find a reading series for children that portrayed males and females in non-stereotyped roles. They could not find any series that was acceptable (Jacobs and Eaton, 1972). These findings are confirmed in studies by the National Foundation for Improvement in Education, in which images of boys and girls are examined. Girls are shown in domestic roles, caring for pets and little brothers; they are encouraged to make themselves look attractive, while success for boys is pictured in terms of independence (Weitzman and Rizzo, 1974). Images of adult women reflect the roles of housewife and mother, presented in an artificial way, but indicating this as the ideal role for women. Similarly, in a study on pre-school picture books, it was found that they present an over-simplified and stereotyped image which presents a very narrow view of society (Weitzman and Rizzo, 1974). Graebner (1972) reports some changes in newer books, showing more women in careers (but stereotyped ones), but men still appear overwhelmingly in biographical stories and they dominate the illustrations and the story texts.

Secondary school textbooks are no better. Trecker (1973) reports that so far as the secondary school curriculum is concerned, humanity is masculine. U.S. History textbooks constantly refer to men, and biographies and stories read by children are all based on the assumption that men lead more interesting lives.

Much time and attention has been given to sex typing in reading books for children, but relatively little to their mathematics texts. This is probably due to the fact that it is assumed that numbers cannot be sex-typed. It is interesting, therefore, that the few studies that have been done show that the mathematics textbooks used by both elementary and high school students do support a stereotyped view of women's role in society. In a study of 2nd, 4th and 6th grade mathematics texts, Jay (1973) found evidence that the mathematics texts used by the children were stereotyped. In a follow-up study it was reported that in elementary mathematics texts, twice as many items were identified as masculine than feminine. Even more striking were the activities in which the boys engaged, as opposed to the girls. Boys are shown to be active, earning money, participating in sports, engaging in inquisitive and exploratory play. Girls play with dolls,

read books, and practice the piano. Men are fathers who earn money by working and engage in leisure activities like fishing, camping and hunting. Women are mothers who sew, make fudge, and seem to spend much of their leisure time shopping or marketing and spending the money fathers earn. Jay and Smenke (1975) suggest some problems could be rewritten with a deliberate attempt to contradict the stereotype, e.g., show boys cooking, girls working, and so forth.

Similarly, Federbush (1974) reports that mathematics textbooks portray boys as active, girls as passive. Where girls are shown to be active, they are doing typically feminine things like jumping rope or going to the store. The "New Math" textbooks are no better. People are always being put into sets by sex. Groups of boys are divided by activities (football players vs. baseball players) while girls are grouped by the color of their eyes and the length of their hair. Female mathematicians are not included when the texts give biographies of famous people.

Even algebra textbooks are not immune to stereotyping. Rogers (1975) examined eight algebra textbooks widely used. Women and their activities were pictured as dull and insignificant; they rarely appeared in career situations. Men, on the other hand, are pictured as alert, active, and more scientific. Women were shown as social. Rogers points out that women excel in "sitting", an activity around which a great many problems are constructed. Some mathematics texts even show females as mathematically incompetent. Standardized tests show these biases as well; the Scholastic Aptitude Test-Mathematics (SAT-M) pictured men with power and influence and referred to girls in connection with mothers and children.

Science texts show similar stereotypes. Even science texts that purport to be non-biased are accompanied by illustrations of only boys using the lab equipment (Trecker, 1973). Boys control the action (a boy rides his bike); girls watch action (a girl watches a balloon float). Adult women are almost never shown in scientific roles. Gaetano (1966) reported that in the upper grades, where tentative vocational orientation begins, males predominate in 16 of 18 texts.

Although it certainly appears that females get little reinforcement for mathematical or scientific interests from texts, unfortunately, one cannot readily assess the impact of these cultural messages upon individuals. A study by Plost and Rosen (1974) does suggest that the same-sex model in media presentations may be a very salient factor for the development of career preference. It would certainly be valuable to study the impact of non-sexist texts upon the achievement of girls in mathematics.

## Tests

Most Mathematics tests contain more biases in favor of males. In a study of the items on the Scholastic Aptitude Test-Mathematics (SAT-M), Donlon (1971) reported that the items heavily favored males. Only two of the items favored females. Analysis of items by content led to the conclusion that the approximate 40 point difference between the sexes on this test is a function of the content formula.

In a study, "Women and Educational Testing", Tittle (1974) found that tests were biased for males. She points out that there are two ways in which discrimination in testing can occur: first, reinforcement of sex role stereotypes and, second, restriction of individual choice in selective bias of test content and user materials.

Strassberg-Rosenberg and Donlon (1975) report that the tests reflect more items relating to things rather than people and that the SAT-M Geometry and regular Mathematics are most biased in favor of boys. Ekstrom, Donlon, and Lockheed (1976) analyzed the California Achievement Test (Level 5, Form A), Iowa Tests of Basic Skills (Levels 11 and 14, Form 6), Metropolitan Achievement Test (Grade level 12, Form F) and Sequential Tests of Educational Progress (Grade 10, Series II). Male nouns and pronouns outnumber female ones in the language of these tests. Ekstrom et al. did not find, however, that items dealing with people were significantly easier for girls than neutral items. Similarly, Donlon, Ekstrom and Lockheed (1976) affirmed the masculine bias in content in four standardized tests.

There is some controversy over whether the wording of mathematics problems affects girls' ability to do those problems. Mullis (1975) suggested that girls have difficulty with word problems in general, not especially related to the sexist content or wording of the problems. In a study of 6th graders, King and Blount (1975) found that there was an interaction effect operating in favor of problems appropriate to opposite sex rather than the subject's own sex, but other studies show that girls relate better to problems more related to their own experiences.

Leder (1974) suggests that both boys and girls prefer doing mathematics problems which are more related to their experiences, although no attempt was made to relate this to performance on those problems. Milton (1959) found that when the characteristics of a problem are altered to make them less masculine, sex differences in problem solving ability were reduced. When the content is very feminine, males no longer outperform females.

In a study using college students, Graf and Riddell (1972) found that it takes females longer to do a problem with a stock market setting than the identical problem in the context of buying yard goods at a fabric store. Graf and Riddell suggest that sex-role stereotypes lead girls to perceive problems in a masculine context to be more difficult, and this perception affects the speed with which problems are solved. They suggest girls may do better on a power test rather than a timed test.

Studies are needed that manipulate various features of the item content, context, and the factors to determine the degree to which test performance is affected by such factors. Test performance is, however, a less crucial issue than course-taking.

## 4. Attitudes, Self-Confidence, and Values

Behavior is influenced by many factors. Although behaviors may be specific to situations, the attitudes and values a person holds are believed to be relatively constant and are the forces that motivate behavior. To understand why a person behaves a certain way in a given situation, one must first understand the person's attitude toward the situation, the person's self-concept as it relates to the situation, and the values the person holds. In the case of mathematics achievement and women one must strive to understand the attitudes women have towards mathematics, towards themselves as learners of mathematics, and the values which help shape these attitudes.

### Attitudes

In college populations sex differences in attitudes towards mathematics have been found (Aiken, 1972; Aiken and Dreger, 1961; Dreger and Aiken, 1957). Males and females, however, do not appear to differ significantly with respect to expressed liking for mathematics or preference for mathematics as compared with other school subjects in the elementary or secondary school years (Aiken, 1970b; 1976; Ernest, 1976; Fox, 1974b; 1975c). In one study, elementary school girls actually reported liking mathematics more than boys did (Stright, 1960). Callahan (1971) found no sex differences on a composite questionnaire of attitudes towards mathematics, but did find sex differences on a few specific items. Girls had a stronger dislike of word problems than boys, but were more likely to report that they enjoyed the challenge presented by a mathematical problem.

Why sex differences are found at the college level, but not before, is unclear. Anttonen (1969) found a rather low correlation between attitudes in elementary school years with those in the high school years. The stability of attitudes is difficult to assess due to problems in the measurement of attitudes, as well as changes in the content of mathematics (Aiken, 1970b).

Whether or not sex differences in attitudes towards mathematics are found at the pre-college level is a function of the definition and measure of attitude. Numerous studies have found sex differences in the expressed usefulness of mathematics as early as grade seven, even in the presence of no sex differences on a general statement of liking for mathematics (Fox, 1975c; Hilton and Berglund, 1974). Thus, in the area of attitudes, perceived usefulness more than the expressed liking for mathematics differentiates between the sexes.

A slightly different measure of attitude is the attitude towards self as a learner of mathematics, or self-confidence. Erlick and LeBold ([illegible]5) found striking sex differences in college students' self-ratings of mathematical, scientific, mechanical and general problem-solving abilities. Fennema and Sherman (1976) found sex differences in self-confidence as a learner of mathematics in high school populations, and Kaminski, Erickson, Ross and Bradfield (1976)

found sex differences in self-concept as early as grade eight.

Ernest (1976) reported that elementary school students were likely to believe that members of their own sex were best at all subjects, but in high school both sexes perceived boys as superior. Fennema (1974b) noted that girls' self-concepts tend to decrease with age and that even when girls are achieving better than boys in mathematics they tend to rank themselves lower in ability. Levine (1976) reports that guidance counselors notice that even when girls earn good grades in mathematics classes they do not perceive themselves as competent.

The finding of sex differences in self-confidence in mathematics is consistent with general findings on sex differences in self-confidence. Although Maccoby and Jacklin (1974) concluded that there were no significant sex differences in self-confidence, numerous studies do find sex differences with respect to particular tasks. Astin, Harway, and McNamara (1976) said that men rate themselves higher than women on academic achievement tasks while women rate themselves higher than men on measures of social competence. In general men and women tend to value men's efforts above those of women, especially in fields that are considered male domains (Etaugh and Rose, 1975; Goldberg, 1968; Mischel, 1974; Henken, Unger and Aronow, 1976; Pheterson, Kusler, and Goldberg, 1971). Deaux (1976) has found that when men suceed they attribute their success to skill whereas women assume they were lucky, not skillful. Women are more likely to seek out activities they perceive as requiring luck rather than skills. Thus, it is not surprising that self-confidence with respect to mathematics differs for males and females.

## Attitudes and Achievement

Attitudes towards mathematics, as measured by expressed liking on more complex questionnaires are generally found to have a low, but significant, correlation with achievement in mathematics at the elementary, secondary, college, and post-graduate levels (Aiken, 1963; 1970a; 1970b; 1976; Aiken and Dreger, 1961; Anttonen, 1969). An interaction between sex, and attitude towards matnematics, with achievement has been suggested by Aiken (1970b; 1976). Crestantrello (1962) found this to be true for college sophomores. Behr (1973) found that mathematics grades were more predictable from attitudes for girls than boys. Jackson (1968) concluded that very positive or very negative attitudes affect achievement, whereas in the middle ranges of attitudes, aptitude is more potent than attitude for predicting achievement.

Studies using expressed liking of mathematics as the attitude measure do not show the clear relationship of attitude to achievement that is found in studies using other measures of attitude. The importance of the perceived usefulness of mathematics was discussed in a previous section. Self-confidence as a learner of mathematics also appears to be directly related to course-taking and achievement for girls.

Sherman and Fennema (1976) found a strong relationship between self-confidence as a learner of mathematics and intent to take advanced courses in high school. In a longitudinal study of 500 students in the junior and senior high school, Kaminski, et al. (1976) found that males and females who had high self-confidence with respect to mathematics did take twelfth grade mathematics courses. Boys with moderate self-concepts with respect to mathematics also took the course. Girls with medium self-concepts tended to follow the pattern of girls who had low self-concepts and not take courses beyond the required level.

It seems likely that a sense of general well-being or self-confidence is necessary but not sufficient for achievement in most academic endeavors (Adams and Fitts, 1972). The exact relationship of self-image to behavior and attitudes is not well known (Harway, et al., 1976).

A general measure of self-esteem appears to be less clearly related to achievement in mathematics than do measures of self-confidence with respect specifically to mathematics. Farley (1968) found no relationship between general self-concepts and girls' decisions to elect or not to elect a tenth grade mathematics course. Cleveland and Bosworth (1967) found sixth grade girls who achieved well in mathematics scored lower on a measure of personal worth than those who achieved less well in arithmetic. Solano (1976), however, found no sex differences in general self-concepts of mathematically gifted adolescent boys and girls. The gifted students had a somewhat more positive self-image than did a random group of adolescents. Helson (1971) found that women mathematicians had high self-esteem.

With respect to attitudes towards mathematics, we can conclude that the perceived usefulness of mathematics and self-confidence as a learner of mathematics are attitudes which reflect strong sex differences, and appear to be related to the differential achievement of the sexes as well as to the labeling of mathematics as a male domain. In terms of expressed liking for mathematics only the very extremes of feeling seem to be related to mathematics achievement and sex differences.

## Math-anxiety

The extreme of very negative attitude towards mathematics has been called math-anxiety or mathemaphobia. Gough (1954) says the concept is self-defining and Tobias (1976) has described

it as an "I can't" syndrome applied to all situations involving mathematics. The concept is intriguing, but at present research evidence about the nature and extent of the problem is meager.

The construct of a specific anxiety about numbers has been validated. Dreger and Aiken (1957) developed a measure of number anxiety that had a low correlation with total scale scores for the Taylor Manifest Anxiety Scale. A second instrument, the Mathematics Anxiety Rating Scale (MARS), has been developed and appears to be a measure of anxiety with respect to mathematics that is independent of measures of general anxiety (Richardson and Suinn, 1972; Suinn, Edic, Nicoletti, and Spinelli, 1972).

The evidence for sex differences with respect to math-anxiety is unclear. Maccoby and Jacklin (1974) noted that general anxiety appears to be more common among females, but Sarason and Winkel (1966) suggest that girls score higher on measures of anxiety because they are more willing to admit their fears. Carey (1958) noted that men are less likely to admit a dislike of problem-solving because they equate it with a denial of masculinity.

Dreger and Aiken (1957) reported no sex differences on a measure of math-anxiety among college students in basic mathematics courses. They found 35 percent of these students were math-anxious. Since they did not comment on the proportion of males and females in basic versus advanced courses, their findings are not totally convincing. Tobias (1976) reports more females than males enrolling in the math-anxiety clinic at Wesleyan. Again, there is the question of whether or not females are more afflicted with the problem, or simply more willing to admit it.

The fact that more women than men avoid mathematics courses in high school and college might be taken as indirect evidence of greater math-anxiety among females than males. Tobias says course avoidance is a classic symptom of anxiety. A rival hypothesis is that math-anxious males are more likely to pursue the courses than math-anxious females, because they perceive the courses as more useful or even unavoidable. Perhaps math-anxious males are more willing to pursue careers and majors that require some mathematical training than are math-anxious females. The relationship of perceived usefulness and career choice to math-anxiety is not known. A study is needed to establish the base rates and developmental history of this phobia.

Anecdotal accounts suggest that the onset of anxiety can occur as early as third grade and as late as graduate school levels. It is logical to suppose that the early adolescent years are particularly critical, as that is the time when sex differences seem to begin to be found on tests and the point at which mathematics becomes more abstract. It would be relatively simple to test this hypothesis.

Math-anxiety can apparently be reduced or alleviated by therapy (Aiken, 1970b, 1976; Hyman, 1973; Tobias, 1976). It may, however,

be possible to reduce the instances of math-anxiety by preventive techniques, such as the elimination of the sex-role stereotyping of mathematics as a male domain. The extent to which math-anxiety is fostered by the perception of conflict between competence in mathematics and femininity is not documented. Although anxiety about mathematics may not in every case be a direct result of sex-role socialization conflicts, it is likely that the sex-typing of mathematics as a male domain by parents, teachers, and peers results in the acceptance of math-anxiety in females as inevitable or irrelevant to their development. As long as competence in mathematics is not perceived as useful or necessary by society, girls are likely to get little help to overcome their fears.

Dweck and Bush (1976) have used the term "learned helplessness" to describe the state in which girls may become conditioned to failure. Although they do not cite specific research with respect to mathematics, the parallel is clear. Girls learn to attribute their failure to their lack of ability, while boys attribute failure to factors beyond their control. Thus, girls learn to avoid situations in which they might fail, while boys learn to strive harder.

Unfortunately much of the research in sex-role socialization does not specifically follow the development of mathematical competence; and conversely, much of the research on sex differences in mathematics has not used a longitudinal and developmental approach. Therefore, the data on the influences of significant others upon attitudes towards mathematics, perceived usefulness of mathematics, and self-confidence as a learner of mathematics are sketchy.

## Parental Influences on Attitudes

Since attitudes, particularly very negative attitudes, do appear to be related to achievement and course-taking, it is important to understand how attitudes are influenced by others.

In a study of college students, Poffenberger and Norton (1959) found that liking for mathematics was significantly related to students' perceptions of fathers' liking for mathematics and fathers' expectations for the students' earning a grade of A for males and females combined. There was no separate analysis by sex. (The positive attitude group, however, was 48 percent female, whereas the negative attitude group was 64 percent female.)

A study by Aiken (1972) found that perceptions of the fathers' liking of mathematics, having been good mathematics students, using mathematics in their jobs, and their professional occupation status were significantly related to positive attitudes of boys, but not girls. For both boys and girls the perception of the entire family as liking mathematics was related to positive student attitudes.

Burbank (1968), however, found no significant correlation between fathers' attitudes and attitudes of either sons or daughters. Hill (1967) also found no relationship for fathers and sons. In

the Hill study, one very significant effect was found. Fathers who sex-typed mathematics as masculine rather than feminine had higher expectations of their sons' performances than did fathers who sex-typed mathematics less. In general, the fathers who had masculine expectations for their sons had higher expectations for mathematics performance. Alas, Hill did not study girls, and no similar study of the effect of sex-typing and expectations for daughters was found.

Fennema and Sherman (1976) found sex differences in students' reports of fathers' perceptions of their offspring as a learner of mathematics in schools where there were also sex differences in achievement and in perception of the usefulness of mathematics. Perceptions of parental attitudes were also significantly related to the intent to take advanced mathematics courses in high school (Sherman and Fennema, 1976).

It appears that perceptions of fathers' expectations, but not fathers' attitudes or professions are related to achievement and course-taking for girls. Presumably, fathers who sex-type mathematics as a male domain are less likely to hold and convey high expectations for their daughters.

The research on the relationship of maternal attitudes and expectations to the attitudes of sons and daughters yields a slightly different picture. Poffenberger and Norton (1956; 1959) found that college students' perception of mothers' expectations for a high grade in mathematics, but not perception of the mothers' liking for mathematics, was significantly related to positive attitudes for males and females. Aiken (1972), however, found the perceptions of the mothers' liking for mathematics and mothers' level of education (having graduated from high school) were related to positive attituded in girls, but not boys. For boys, however, the perception of mothers' earning high grades in mathematics was related to positive attitudes. Burbank (1968) found a relationship between maternal attitudes and attitudes of both sons and daughters.

Fennema and Sherman (1976) found sex differences in perceptions of mothers' perceptions of student as a learner of mathematics in schools where there were also sex differences in achievement and perceived usefulness of mathematics. Intent to take advanced coursework was also related to students' perceptions of maternal perceptions (Sherman and Fennema, 1976).

Thus, for girls, perceptions of parental expectations and maternal attitudes towards mathematics appear to be important. It would be interesting to know to what extent the sex-typing of mathematics as a male domain influences maternal expectations for girls. Perhaps mothers who had a favorable attitude towards mathematics themselves are less likely to sex-type mathematics and, thus, more likely to have high expectations for daughters.

There appears to be no single study which has measured all the relevant dimensions to unravel the relationship of parental sex-typing, expectations, encouragement and child behavior. It would

be helpful to have a study which measured all the following variables.

1. Attitudes towards mathematics for both parents (including perceived usefulness of mathematics for daughter)
2. Parental expectations for child's performance
3. Parental sex-typing of mathematics
4. Parental estimate of own ability, and report of parent achievement
5. Education and occupation of parents
6. Student's attitudes towards mathematics (including perceived usefulness of mathematics)
7. Students' self-expectations
8. Students' self-rating of ability
9. Students' sex-typing of mathematics
10. Students' career aspirations
11. Students' actual achievement
12. Students' aptitude
13. Students' perceptions of parents' attitudes
14. Students' perceptions of parents' expectations
15. Students' perceptions of parents' ability
16. Students' expected and actual course taking grades 9 - 12.

If all of these variables were assessed, it would provide some interesting insight into the impact of parents upon the mathematics achievement of their children. In too many studies the child's perceptions of parental views have been interpreted as though they are the actual views of the parents or the construct attitude is not clearly defined, and measures of achievement vary. The interplay of all these variables needs to be explained more carefully.

## Teacher Influences on Attitudes

It is generally believed that the child's most formative years in terms of cognitive abilities, personality, values, and sex-role identification are spent in the home, not in the school. As the child moves into adolescence, the peer group begins to take on a stronger socializing influence than even the family (Coleman, 1961). Thus, teachers and the school system, by themselves, may have only moderate influence upon the child. Their power to influence may be strengthened or weakened in accordance with the degree to which they work with or against these other more primary socializers.

Thus, it is not surprising that Poffenberger and Norton (1959) concluded that teachers have relatively little influence on the development of students' attitudes towards mathematics and that self-concepts with regard to mathematics are well established by the influences of the parents in the pre- and early school years. Their own data, however, do not seem to fully warrant this conclusion. Retrospective reports of students did indicate some influence of previous year teachers on attitudes.

In a review of the literature in 1970, Aiken (1970b) noted the problem of inferring causal relationships between teacher attitudes or behaviors and student attitudes and achievement. In a study in 1972, Aiken concluded that teacher attitudes do influence student sttitudes. Student perceptions of mathematics teachers as negative and demanding were significantly related to negative attitudes in both sexes. Positive feelings towards mathematics were associated with positive feelings towards the teacher for boys more often than for girls. Girls appear to be able to like their mathematics teachers while simultaneously rejecting the subject (Aiken, 1972; Poffenberger and Norton, 1959).

In a review article on attitudes and mathematics in 1976, Aiken concluded that the belief that teachers' attitudes affect student attitudes towards mathematics has been difficult to confirm. Starkey (1971) concluded that the effect of teacher attitude and behavior on student attitudes and behaviors vary greatly from teacher to teacher, and student to student.

Although there is little evidence, it does seem that teacher beliefs about sex differences in mathematics, and their supportive or non-supportive behaviors, can affect girls' achievements and attitudes in mathematics. (Additional evidence on the importance of teachers is discussed in the section on significant others.) Ernest (1976) reported that 41 percent of a small sample of teachers (primarily female), believed that boys do better in mathematics than girls, while no teacher felt that girls did better than boys. Clearly, this type of attitude, which is different from the teacher attitudes typically studied, is likely to influence behavior. In the Wisconsin study, the perceptions of teachers' views of the students as learners of mathematics were significantly related to intent to take additional courses, but were not obviously related to sex-differences of the achievement tests (Fennema and Sherman, 1976; Sherman and Fennema, 1976).

A study by Bean (1976) reported differential interactions between teachers and male and female students in high school mathematics classrooms. Teachers initiated fewer interactions with females than males. Females also initiated fewer contacts with the teacher than males. Perhaps some of this differential interaction is related to teachers' and pupils' perceptions. In a study of twelve schools that had twice the natural percentage of girls enrolled in advanced placement calculus and physics courses, Casserly (1975) found that teachers of these courses had actively recruited girls for the courses. They also expected and encouraged high levels of achievement. Self-reports of women mathematicians emphasize the impact of an encouraging teacher (Luchins, 1976). Fox (1974b, 1976b) found that some teachers actually ridiculed girls who tried to accelerate their progress in school. Solano (1976) found that teacher stereotypes of mathematically gifted girls were considerably less favorable than teacher stereotypes of mathematically gifted boys.

Although the general literature on attitudes and expectations of teachers has failed to show conclusive results (Aiken, 1976; Rosenthan and Jacobson, 1968), it seems that much more research is needed, particularly research relating specifically to sex-role stereotypign by teachers. Even women mathematics teachers may not be providing girls with the same encouragement as they do boys. Elementary school teachers who are anxious about mathematics themselves may be more potent negative modesl for girls than for boys.

## Peer Influences on Attitudes

Teacher's attitudes may be less potent than those of peers. Unfortunately, there are relatively little data. Shapiro (1962) found that peer attitudes in elementary school are determiners of attitudes toward mathematics, especially for girls. In a study of adolescents, Poffenberger and Norton (1959) argued that peer-group influence helped to reinforce already developed attitudes. No discussion of sex differences was provided. Anecdotal accounts of gifted girls suggest that girls fear rejection by peers if they appear too bright in mathematics. Solano (1976) found that student stereotypes of mathematically gifted girls were more negative than those of gifted boys.

It seems likely that social class may also related to peer perceptions. Entwisle and Greenberger (1972) found blue-collar adolescent males to be the most liberal towards career achievement of women. The taking of advanced mathematics courses may be more normative for boys and girls in schools that have high percentages of college bound populations. An analysis of already existing data banks on high school students who apply to colleges and take examinations, such as the Scholastic Aptitude Tests, might provide some clarification of this issue.

## Other School-Related Influences on Attitudes

Clearly, the stereotype of mathematics as a male domain is prevalent in media, textbooks, and tests. There is no research, however, which shows how these factors influence girls' attitudes towards mathematics as a male domain, or their actual achievement and course-taking. Continual exposure to jokes about women's poor mathematical skills, and the portrayal of mathematics as a male domain do shape girls' attitudes. Girls who do like mathematics must feel some sense of conflict in expressing such unfeminine interests.

In general, the reports suggest that neither curriculum nor specific teaching strategies have significant effexts on attitudes. Aiken (1970, 1975) reviewed the literature on attitude change as a result of specific curriculum, such as SMSG, UICSM, Ball State, etc., specific teaching strategies, or the use of motivational devices, such as calculators and computers. Many of the studies do not analyze the attitude change separately for girls and boys. Lack of significant

findings in some studies may be due in part to failure to control for other relevent factors. For example, Devine (1967) found that teacher experience interacted with instructional approach. When the teacher of a programmed course was inexperienced, achievement was equal to a teacher-directed comparison class, but negative attitudes towards mathematics developed. When the teacher was experienced, achievement was lower, but attitudes were not affected.

Aiken's (1970b) review of the literature on attitudes towards mathematics reported that studies of ability-grouping had not found this practice to have significant effect on attitudes or achievement. More recent studies (Brody, 1976; Casserly, 1975; Fox, 1975b; 1975e; 1976a) suggest that ability-grouping may affect course-taking behavior of girls. It seems likely from these studies that ability-grouping alone is not enough. Other factors that may interact with ability-grouping for girls are:

1. pace of instruction and level of content
2. duration of the tracking, and some social aspects of the situation
3. number of girls in the program
4. teacher characteristics and behaviors
5. parental and peer support
6. attitudes, interests, and values of the student

What is needed are studies which identify teaching strategies or curricula that affect girls' attitudes towards the usefulness of mathematics and their willingness to take advanced courses. A study at Wellesley (Schaefer, 1976) is experimenting with a special mathematics curriculum for freshmen that is radically different from typical mathematics programs. In this study, the importance of social and aesthetic interests of women has been considered. The evaluation is not completed. If this proves successful, similar studies with younger populations would seem warranted.

## Changing Attitudes and Behaviors

In a small pilot study of a career education and skills course in mathematics, Fox (1976d) found that girls' attitudes towards mathematics became more positive after the program. In this program, emphasis was placed upon the application of mathematics to social problems and art. Farley (1968) found that girls who were not given a choice with respect to tenth grade mathematics became more favorable towards mathematics than girls who had the opportunity to elect or not elect the course. In general, however, it may be more efficient and effective to modify behavior directly rather than attitudes (McGuire, 1969).

Mathematically gifted seventh-grade girls who participated in an accelerated algebra program were significantly more accelerated in their mathematics progress by the end of the tenth grade than were the girls in a control group. The experimental girls were not, however, more favorable towards mathematics or mathematical careers after the special class, or three years later (Fox, 1976e; 1976f). Thus, changing the behavior had little effect upon the expressed liking for mathematics, but still accomplished the more important goal of modifying course-taking behavior.

Carey (1957) found significant sex differences in attitudes towards problem-solving as well as performance on problem-solving tasks. After intervention, a discussion session to improve attitudes, problem-solving behavior improved for the women but not the men. Attitudes, however, did not change for either sex.

A limitation of the studies of attitude change is that they have in general not focused on specific attitudes of perceived usefulness of mathematics and perception of self as a learner of mathematics. Perhaps when behavior is changed, it is these attitudes, not expressed liking for mathematics, problem-solving, or specific career choices, that are affected. More research is needed.

Values

On the basis of a review of the literature on several measures of social orientation such as empathy, response to social reinforcements, proficiency in imitating models, and amount of time spent in interactive social play, Maccoby and Jacklin (1974) concluded that the belief that girls are more "social" than boys was unfounded. They did not cite, however, the studies of social (altruistic) value orientation versus theoretical (intellectual) value orientation among adolescents and adults.

Numerous studies report sex differences on the value scales of the Allport-Vernon-Lindzey Study of Values. Indeed, in most studies women score higher than men on the social, aesthetic, and religious scales and lower on the theoretical, economic, and political scales (Allport, Vernon, Lindzey, 1970). Differential value profiles are related to educational and occupational choice. For example, the profiles for female medical students are distinctly different from female graduate students in business or nursing, or art students.

High scores on the theoretical scale are associated with interests in science and mathematics. MacKinnon (1962) found high scores on both theoretical and aesthetic scales to be typical of creative mathematicians. Studies at the Johns Hopkins University have found high theoretical scale scores to be characteristic of mathematically precocious adolescent males (Fox, 1976g; Fox and

Denham, 1974). Mathematically gifted girls were more likely than the boys to have high social or aesthetic scale scores. Differential values seemed to account for some of the observed sex differences in interest in mathematical acceleration and careers among academically talented youth.

Although it is possible to measure values of gifted students as early as grade seven (Linsenmeier, 1976), there are no studies of the development of value orientations as measured by the Study of Values from early adolescence to adulthood. One study of stability of values for college students between the freshman year and graduation found the following: First, values appeared to be very stable over the college years, although there was some tendency for aesthetic value scores to increase and religious value scores to decrease; second, values of students in specific majors were similar; and third, individual values shifted slightly to become even more like the major group profile (Feldman and Newcomb, 1970). These results are consistent with other studies of personality which suggest that personality formation occurs early and is not highly flexible and fluid. Value orientations are not likely to be as amenable to change as are attitudes.

Influences Upon the Development of Values

Few studies have examined the relationship between parents and children on the Study of Values. Fisher (1948) studied college students and their parents and found that students' values profiles were more similar to their same-sex than opposite-sex parent. Gray and Klaus (1956) found that both college males and females had profiles more like their mothers than their fathers. Females were more like both parents than were males. In a study of mathematically gifted adolescents Pyryt (1976) found that profiles of boys were more like their parents' profiles than were the profiles of girls. Although boys' profiles were more like their fathers than were girls, girls were not significantly more like their mothers than the boys. A very small number of girls and boys had become accelerated in mathematics. These students had profiles like their opposite-sex parent.

It is interesting that the profiles of mathematically gifted females in the Pyryt study were less like those of either parent than were females in the college samples. The small number of students in the study makes generalization difficult.

It is surprising that there is so little research on the devleopment of values and the relationship of values to learning environments. Perhaps values have seemed to be too personal or too political to be explored. It seems logical to assume that values are developed as a result of learning rather than by purely biological maturation. The fact that males and females tend to develop

rather different value profiles suggests that the impact of strong but subtle conditioning. It would seem important to understand more about the process of value development and its reinforcement by family, schools, and peers.

Values and Educational Strategies

The social value scale of the Study of Values is a measure of altruistic concern for people rather than a measure of sociability. Persons who score high on this value scale may find mathematics classes that emphasize the theoretical and abstract nature of mathematics less interesting than courses which emphasize the application of mathematics.

To date, there are no reports of programs that have studied value-treatment interaction in science and mathematics instruction. Sex differences in mathematics are even more likely to be directly related to value/interest correlates of sex-identity than to innate biological/cognitive correlates. Research of this type is sorely needed. Perhaps boys who have low theoretical values also underachieve in mathematics programs that have theoretical orientations. Instructional strategies that recognize and adapt to differing values and interests might foster greater achievement for both boys and girls.

Knowledge of the value orientations of students and their parents might be useful information for identifying girls and boys who may not enjoy traditional mathematics classes, or who will receive little parental support. Three program implications come to mind. Girls who have low theoretical value orientations may respond differentially from girls with high theoretical orientation to various kinds of mathematical experiences. The former might be more interested in learning mathematics if it were taught in ways to emphasize social or aesthetic applications, as the program at Wellesley does. Second, girls who have moderate or high theoretical orientations may be those who are most predisposed to respond to career education efforts and encouragement to develop within more traditionally scientific areas. Third, parents whose values are not highly theoretical, but who have mathematically talented children, might benefit from family counseling programs aimed at increasing their recognition, acceptance and nuturance of mathematical or scientific talent and interests of their children.

## 5. Educational Policies and Practices

The very fact that fewer mathematics courses are required in high school than are courses in English, or even sometimes social

studies or physical education, reflects the belief that mathematical competence may not be necessary for everyone. Although it does seem true that not all students are capable of mastering advanced courses, such as calculus, perhaps girls who are capable of learning it should not be allowed to omit it from their programs. If four years of high school mathematics were required, perhaps more girls would take the advanced courses. In a comparison of girls in schools that required a third year mathematics course with those in a school that did not, Farley (1968) found that the girls who were forced to take the course showed improved attitudes towards mathematics, whereas girls who elected the course did not. A seventh grade intervention program for gifted girls also had significant results (Fox, 1976e). Girls who became accelerated in mathematics by one year as a result of a special summer program continued to accelerate by the end of the tenth grade. A control group of girls was not equally accelerated. Yet the control boys had become accelerated, presumably by virtue of their own initiative and encouragement from parents and teachers.

Although it may be difficult to mandate the advanced courses, current teacher and counselor practices should clearly be changed. Teachers and counselors should make a concerted effort to identify and encourage girls who have the ability to take the courses. Perhaps the implementation of accelerated mathematics programs in the elementary and junior high school would foster greater course-taking in the high school and college years.

Programs for the gifted, especially the mathematically gifted, do appear to make a difference. Casserly (1975) noted that many of the girls enrolled in the advanced calculus and science courses felt that their induction into special gifted or academic tracks as early as grade four had been a major determiner in their taking the advanced courses. Special accelerated mathematics programs initiated in several school systems in Maryland appear to be highly successful in accelerating the mathematics progress of girls, particularly when the program is integrated into the basic school program (Brody, 1976; Fox, 1975e; 1976f).

The acceptance of programs for the gifted at all school levels would also be useful for identifying girls for whom career education programs that emphasize professional career areas would be most appropriate. Once girls are identified as gifted and tracked into advanced and accelerated programs, they may be likely to raise their self-expectations, and expectations of their parents, teachers, and counselors may be modified. Such programs may relieve girls of the pressure to defend a choice to pursue the advanced courses.

Special programs and advanced courses are likely to be most beneficial to girls if the number of girls does not become too small. This has been noted by both Casserly (1975) and Fox (1974a; 1976a). The exact number of girls needed to form a "critical mass" is not known.

Nor do all girls shy away from being the only girl in an otherwise all-male class in school. The inclusion of a sizeable number of girls, however, does seem to make a difference. Fox (1974b; 1976c) found that girls were far more eager to participate in all-girls accelerated classes than in ones where there were very few girls. In mixed-sex classes, if one girl drops out, others may soon follow (Angel, 1975). Casserly (1975) noted that girls in advanced placement courses cited the need for a female peer cohort for moral support.

Whether or not all-girl schools or classes promote greater achievement among girls than mixed-sex groups is not clear. Hurley (1964, 1965) found that scores on several achievement measures, including arithmetical concepts, improved for fifth grade girls after one year of a same-sex class, but no increases were found after the second year. The reverse was true of males in all-boy classes. Although teachers felt there had been significant changes in behavior of both boys and girls in same-sex classes as compared with mixed-sex ones, especially in terms of less sex role stereotypic behavior, the boys, but not the girls, liked the sex-segregation. Although Fox (1974b; 1976c) found girls more willing to participate in all-girl accelerated mathematics classes than in mixed-sex ones, achievement was not notably better. This may have been, in part, because of the different selection procedures used. The girls in the all-girl class were less highly selected than in mixed-sex comparison groups. Husbands (1972) has argued against sex segregation on the grounds that separate rarely can be separate and equal. There is a danger that science and mathematics courses in all-girl schools will be of lower caliber than in all-male schools. The data are not strong enough to warrant mandating all-girl classes as part of a regular program. It may be, however, that for career education purposes and special accelerative or remedial programs, all-girl classes will be more successful than mixed-sex ones. For example, remedial mathematics programs at the college level may be more successful when all-female groups are employed (MacDonald, 1976). Further research is indicated, but the restraints of Title IX may be a problem. Although Title IX may inhibit some research efforts, by and large it will probably reduce some sexist practices, such as all-male schools that emphasize science and mathematics.

The sex of the teacher is probably totally irrelevant to the success of girls in regular or special programs. The attitudes of the teacher may however, be very important as found by Casserly (1975). The continued use of textbooks and tests that reflect sexist bias, however, should not be tolerated. Even though there is a lack of direct research evidence to support the negative consequences of the use of such materials, the implications of the bulk of the research is that the prevailing stereotype of mathematics as a male domain inhibits achievement of females and must be discouraged.

A final educational practice that works against the achievement of women in mathematics is the failure of many, if not most,

educational institutions to provide career education programs and counseling services that seriously consider the special needs of women. Some general insight into the problems of sex discrimination of schools and counseling is provided in a series of reports.

## Conclusions and Recommendations

There are typically three general hypotheses offered as explanations for male superiority in mathematics (Aiken, 1976). The first is that biology is destiny and genetics not interests, effort, and cultural expectations and conditioning account for sex differences. The second is that psychological masculinity is required for excellence in mathematics. The third is that the sex-typing of mathematics as masculine results in differential expectations and socialization practices for boys and girls with respect to achievement in mathematics. The first hypothesis was not addressed in this review of the literature. The second hypothesis was not strongly supported by the literature. The third hypothesis does appear to be strongly supported by the existing research.

### Conclusions

Despite the lack of longitudinal studies, particularly ones that assess the multi-factor nature of the issue, and the confusion of terms, the bulk of the evidence on sex role socialization and mathematics achievement points in the same general direction. Women achieve less in mathematics and science than men in terms of careers and on tests partly, if not totally, because that is what society expects and encourages. Subtle and not so subtle messages that mathematics is a male domain are given to girls and women by parents, peers, educators, and society at large. Although the penalties for trespassing into the male domain are not so severe as to threaten life or limb, they are harsh enough to deter all but the most intrepid souls. Society encourages girls to believe that there is a conflict between the ideal feminine role and adult achievement in careers, particularly in mathematics and the sciences.

Sex differences in adolescence and adulthood on mathematics tests and career choices appear to result, at least in part, from the differential pursuit of mathematical experiences, notably course taking in the secondary and postsecondary school years. Differences in course-taking for the sexes is both cause and effect. Although sex differences in course-taking may not explain all the observed sex differences on tests of achievement and aptitude, it is by itself a major achievement difference and contributes heavily to differences in adult career options. The reduction or elimination of sex differences in course-taking should lead to decreases in sex

differences on other measures of achievement. Even if course-taking could be made constant, some sex differences might still exist as long as the underlying causes of differential course-taking are left unchanged. To reduce or eliminate sex differences in achievement in mathematics we must first understand the social conditions that foster differential interest in the pursuit of mathematics education.

Two factors that have been found to relate directly to sex differences in course-taking are the perception of the usefulness of mathematics for careers and the support and encouragement from significant others. A third factor that also appears relevant but for which there is less direct research evidence is educational policy and practice. Underlying these three factors and a factor in itself is the perception of mathematics as a male domain.

Perceived Usefulness of Mathematics

Sex differences in the perception of the usefulness of mathematics for adult life has been reported as early as grade seven, the age at which sex differences on achievement measures are typically first found. It is quite likely that these differences actually develop far earlier than grade seven. This does not appear to have been researched. It may be, however, that the perception of the usefulness of mathematics is also influenced by the fact that the nature of mathematics appears to change from arithmetic skills and concepts to more abstract algebraic and geometric concepts at the middle-school years. The pre- and early adolescent years are also those in which the awareness of the social and biological differences between the sexes becomes a crucial force in the development of the child. It is at this point that awareness of sex-role appropriate behavior is likely to become quite potent in influencing behavior. It is then perhaps not coincidental that in adolescence many girls begin to perceive themselves as less competent than boys in mathematics. The perception of the usefulness of mathematics appears to be related to the perception of mathematics as a male domain and to some general beliefs about appropriate careers for women.

Sex differences in career interests and aspirations are found as early as kindergarten. In general girls are less oriented to careers other than homemaker than boys and those who are career-oriented are less likely than boys to be interested in professional careers in mathematics and science. It is not too surprising that girls perceive mathematics as less important for their future than do boys. Some girls who are career oriented may be prematurely self-selecting themselves out of mathematics courses because they are unaware of the true relevance of these courses for the careers in which they are interested. Ignorance at the importance of mathematics for many career areas is perhaps a result of the lack of adequate career education programs in the schools as well as

ignorance or sexism on the part of counselors, teachers and parents. Lack of interest in careers in general seems to be a result of the complex socialization process during which girls learn to perceive a conflict between achievement (careers) and the feminine role of wife and mother despite the fact that the great majority of today's women do indeed spend some time in the labor force. The avoidance of professional and technical careers, particularly in mathematics, is also a result of the lack of available women role models and the stereotype of these career areas as unsocial. Why women appear to be more oriented towards social and aesthetic than theoretical and economic values and careers is not totally clear. Society, particularly the significant others in the child's life, appears to reinforce if not cause these differences.

Support from Significant Others

Girls' decisions to take or not to take advanced courses in mathematics are directly influenced by the advice of significant others. School counselors, alas, are likely to discourage rather than encourage girls to take the courses or to pursue careers in mathematics and science. Although the impact of a non-sexist and inspirational teacher may be greater than the influence of a counselor upon a girl's perception of self-relative to mathematics, teachers are by-and-large likely to believe and reinforce sex-role stereotypes. Girls may be even more sensitive to peer pressure than to the influences of educators. Whether or not their perceptions are accurate, girls seem to believe that excellence in mathematics is socially undesirable. Adolescents do appear to have a more negative image of mathematically talented girls than boys; and males, particularly white middle-class males, do not appear to be supportive of achievement for girls. Girls tend to orient their career aspirations in keeping with their perceptions of what males will tolerate. On the other hand, support from a strong female peer group may enable girls to deal with male peer pressure when selecting courses and careers. Although all-girl classes may be unacceptable under Title IX and not totally acceptable to many girls, the sex-ratio of a mathematics class may be an important factor in girls' willingness to take advanced or accelerated mathematics and science courses. Peers, teachers and counselors may be less potent as changers of attitudes than as reinforcers of attitudes already shaped by the home and society at large. Parents, particularly fathers, play a crucial role, directly and indirectly, in the development of attitudes, career interests and decisions to elect or not elect courses in mathematics.

Although there may be a great deal of similarity in the early socialization experiences of boys and girls, parents often have lower educational expectations for daughters than for sons. Parents have the earliest influence upon the child's concept of self and upon the child's perceptions of sex roles and sex-appropriate behavior.

If parents sex-type mathematics and related activities as masculine this view is communicated to the child. Parents do tend to reinforce sex-role stereotypes in their choice of toys and in their greater acceptance of low levels of achievement in mathematics for daughters than sons. The attitudes and expectations of the father may be more important than those of the mother since the father is more likely to be the parent who exhibits interest in mathematics and science and to whom the daughter turns for help with mathematics homework and advice about course-taking. Mathematical competence appears to be adversely affected by the absence of the father for both sexes. If mothers, however, are strong models for career aspiration and competence in mathematics, they may have a potent effect upon their daughters' attitudes and career orientations. The fact that little girls often perceive women only as homemakers appears to shape their expectations for their own future roles.

Identification with the mother or father does not necessarily exclude or foster competency in mathematics. The nature and importance of a female child's identification with either the father or the mother or the generalized masculine or feminine role with respect to achievement in mathematics are not clear. What seems to be a most promising explanation is that females who have very feminine orientations can succeed in mathematics or pursue a mathematical career if these activities are not perceived by them as incompatible with the feminine role. If the females perceive these activities as masculine then they will succeed less well in them than males unless the females have a masculine or at least androgynous role orientation. The argument that identification with an abstract masculine role by boys promotes an analytical cognitive style does not seem to be supported by evidence. Clearly more research is needed to untangle the issue of masculinity-feminity dimensions of psychological identification and mathematical competence.

There is also surprisingly little direct research on the impact of specific child-rearing practices upon the development of mathematical competence or career orientations in girls. The research suggests that those practices which foster independence and self-reliance also foster competence in mathematics whereas children who are over dependent or who have interfering and demanding mothers may develop discrepant verbal and mathematical abilities. Many of these are based on samples of children selected for their discrepant ability patterns without studying the ability patterns of the parents to control for possible hereditary factors. Studies do indicate, however, that parents of boys are more likely to emphasize achievement, competition and control of feelings while parents of girls stress interpersonal relationships, affection and protection. While it seems logical that these parenting practices will eventually lead to less intellectual risk-taking on the part of girls than boys, there is no direct evidence as to the impact of these practices upon the development of mathematical ability.

Thus while there is some indication that parents are the most important influence upon the development of the child's view of self and the world, the exact dynamics by which the child learns sex-role appropriate behavior from the parents is unclear. Nor is it entirely clear exactly how parents foster or inhibit their child's achievement in mathematics and careers. It does appear, however, that many girls do receive some direct messages from their parents which indicate low expectations for success in mathematics and the pursuit of professional careers. Girls who take the advanced mathematics courses are likely to report encouragement from parents. In general, however, parents may be less supportive of daughters than sons because of their own stereotypes of appropriate feminine activities.

Educational Policies and Practices

Although the perception of the usefulness of mathematics for future careers and the support of significant others are the most directly obvious factors associated with course-taking, educational policies and practices also have some influence. If mathematics course-taking was not optional in the secondary school years, perhaps there would be more girls enrolled in the advanced courses. When the courses are optional more boys than girls take them because the boys are encouraged to see these courses as necessary while the girls are not.

Programs for the academically able student that begin in the elementary school years also appear to promote greater course-taking in mathematics and science at the secondary level. Although "tracking" has become unpopular and only a few states have active programs for the intellectually gifted child, boys who have ability tend to move ahead in mathematics at a more rapid pace than their gifted female cohorts. Perhaps if programs for the academically gifted were widerspread more girls would be encouraged to take the advanced courses in mathematics and science.

Until recently some classes were segregated by sex, such as home economics, shop and mechanical drawing. Such practices helped institutionalize sex-role stereotypes. It is perhaps unfortunate that efforts to eliminate these sexist practices may also eliminate sex-segregation for purposes which might ultimately benefit both boys and girls. For example, special classes in mathematics or science just for girls might be useful for attracting girls who would otherwise avoid these courses for fear of competition with males.

Although there is no evidence that the sex of the teacher is an important factor for achievement or attitudes in regular classes, the fact that teachers of science and mathematics at the secondary level are typically male helps reinforce the image of mathematics as a male domain. The problem is of course circular. It will be difficult to recruit more females for these jobs as long as many women avoid the courses at the high school and college level.

Perhaps the major institutional practice that poses a barrier to women's entry into courses and career areas in mathematics and science is the lack of career education and counseling programs at the elementary, secondary and postsecondary school levels. Even schools which have such programs may upon closer scrutiny discover that these programs still perpetuate the stereotypes of limited career opportunities for women. Also, these programs often do not include female role-models for atypical careers which appear to be a necessary component of an effective career education program.

## Mathematics as a Male Domain

The view of mathematics as a male domain is wide-spread and portrayed in the media, textbooks and tests. Parents, teachers and students all appear, at least for secondary students, to believe that this is true. This belief undoubtedly underlies the foregoing correlates to sex differences in course-taking and achievement in mathematics. Although there is some question as to whether or not females fear success, there is ample evidence that females who wish to appear "feminine" are more comfortable in task-situations that are labeled feminine or at least neutral than in those labeled masculine. The sex-typing of mathematics as a male domain leads to different expectations for boys and girls with respect to success in mathematics classes. These self-perceptions are reinforced by behaviors and expectations of teachers, peers and parents. It is little wonder that girls have less self-confidence than boys as learners of mathematics. This presumably accounts for differences in course-taking when courses become optional.

It has been hypothesized that in some cases the conflict between femininity and competence in mathematics becomes so great that an actual phobia or anxiety about mathematics develops. At present there is little information as to the causes of math-anxiety, but it does appear to exist. Even females who on tests appear to have well-above average aptitude for mathematics are not apparently immune to the phobia. For some adolescent girls this fear may result from an approach-avoidance conflict. They may be attracted to mathematics and related careers by virtue of their abilities and interests yet be too insecure in their own feminine identity to trespass upon male territory or be drawn by conflicting interests to more aesthetic or social pursuits.

The stereotype of the mathematician or scientist as cold and asocial is likely to contribute to this conflict. It does appear that females have more social than theoretical values. It is unfortunate that mathematics is so often taught in such a way as to deemphasize its relationship to the arts and the social sciences. Mathematics class-rooms, particularly at the advanced course level, do indeed appear to be male domains taught by men to largely male classes.

As long as mathematics classes and careers in related areas maintain their masculine mystique the problem is somewhat circular. Girls avoid mathematics classes and careers because they perceive them as male domains; they receive little encouragement to pursue courses and careers in these areas by significant others who also appear to perceive these activities as male. As long as women avoid these courses and careers there will be few role models and the image will remain unchanged. The question of interest is how to break the chain and to reduce the sex-typing of mathematics as a male domain and increase girls' participation in mathematics classes and careers.

## Recommendations for Educational Policies and Practices

To increase girls' participation and achievement in mathematics classes at the secondary and postsecondary school levels, it will be necessary to increase girls' awareness of career opportunities for women and the importance of mathematics to many career areas including homemaker. Girls should also be made aware of the wide variety of careers in business, government, and industry that require mathematical competence and yet have social service components. Appreciation of many avocational activities in art, music, and games might be heightened also by an awareness of their relationship to mathematical concepts.

The most obvious and direct method for fostering an understanding of the relevance of mathematics to careers and life would seem to be through career awareness and education programs at the elementary through postsecondary level. Such programs could be widely varied in nature; they should probably include, however, some exposure to women who have full- or part-time careers related to mathematics and the sciences. Career education programs need not be costly. Many community resources exist for little or no charge and retired persons might serve as volunteers. Such programs should be constructed with a concern for eliminating sexist portrayals for women. Some existing programs should also be carefully evaluated to eliminate "unconscious" sexism. Some standards or guidelines for career education programs need to be developed and disseminated to schools. Some preliminary research and pilot-testing of model programs may be needed before such guidelines can be developed.

Non-sexist career counseling is also needed. In order to provide this some special inservice training for guidance counselors may be needed. It might also be valuable to generate a list of non-sexist vocational interest inventories or guidelines for improving the interpretation of existing instruments for women. Some research is already in progress and should be continued.

The provision of non-sexist career awareness, education, and counseling programs should help dispel some of the negative peer pressure against girls' pursuit of "atypical" careers. The attack on sexism and racism should not be left to chance. Program materials should be designed to directly confront and dispel myths and prejudices. It might also be wise to design such programs, particularly those at the elementary and junior high school levels, in such a way as to elicit parent and classroom teacher involvement so that they will be aware of the purposes and merits of such programs. Parents might be encouraged to serve as volunteer workers in some aspects of career and community education.

Parent-effectiveness training has recently become popular in some areas of the country. Such programs are not typically offered by the public school system. Perhaps public education should offer courses for parents that encompass a wide range of topics including agism, sexism, racism, and the importance of career education and guidance. Many parents may not realize that their attitudes and behaviors, such as the choice of toys and games for their offspring, could influence later learning.

In-service programs for teachers should also be developed. Many teachers may be totally unaware of their own sexist practices or beliefs. Mathematics and science teachers might be encouraged to develop special units that depict the importance of these subjects to broad areas of human endeavor including social service and aesthetic careers and activities. In some cases the practical applications should receive greater emphasis. Courses on statistics, computers and other areas of applied science and mathematics might be developed and offered at the upper elementary and lower secondary level. Teachers should be encouraged to take a critical look at the textbooks they use and the activities and tests they develop. Some general guidelines might be developed and disseminated through teacher organizations such as the National Council of Teachers of Mathematics. Teachers should be advised to specifically identify the boys and girls who appear to have very high aptitude for mathematics and to encourage both girls and boys to pursue courses and careers in this area. If teachers, counselors, and school administrators seek out these students at an early age they may be able to increase the participation of girls in the more advanced courses.

Programs for the gifted and talented child that recognize and facilitate the developments of special talents, such as mathematical ability, may have a positive effect on the course-taking behavior and attitudes of above-average ability girls. Early identification programs would enable schools to identify those girls who have great academic potential at an age where special intervention in the form of career education and special encouragement in mathematics might be most beneficial. The early tracking of these girls into college

bound programs that emphasize the importance of mathematics and science could have a potent positive effect on later achievement and career choice. Concomitant counseling for the parents of such students would also be desirable. For such talented students special after-school or summer courses that combine career education with mathematical skills might be valuable.

Early screening programs to identify gifted students might be integrated with existing programs to identify students who need special remediation in basic subjects. Early and continued identification programs might also seek to locate students who are developing anxieties about their abilities in mathematics. To some extent such programs now exist to identify students who show potential for difficulty in reading. Such efforts should be expanded to include those who show high verbal or reading competency but poor arithmetical skills development. The concept of literacy should be expanded to include quantitative literacy as well. The Right to Read program might be broadened to include the right to learn basic computational skills, arithmetic concepts, and the applications of these to everyday life. At present most school systems require only one or two years of mathematics at the high school level for graduation. Increased requirements and more varied course offerings would also seem desirable for all ability groups.

There has been some concern with the sexism present in media, tests, and textbooks. This concern has by-and-large not been as directed towards mathematics texts and tests as it has been towards basic readers and children's literature. The image of mathematics as a male domain might be greatly reduced if textbooks, tests, and media were less sexist with respect to mathematics. Textbook adoption committees at State and local school system levels should be encouraged to scrutinize the offerings for their sexism as well as racism. Publishers of educational materials should be made aware of the negative aspects of sex-role stereotyping and urged to develop materials that portray women in a variety of roles and as capable and active rather than incompetent and passive and avoid equating mathematical and scientific interests with masculinity. Although it may be impossible to mandate standards for publishers or textbook committees, lists of recommended books and materials as well as informal guidelines might be generated and widely disseminated. Some federal grants might be given for the creation of non-sexist educational materials as models. It is difficult to know how to best influence the media. Federal and private foundations that give grants to public broadcasting efforts could at least be urged to demand non-sexist and non-racist programming. Professional women's organizations might also be encouraged to become more vocal advocates of non-sexist practices.

Professional women's groups such as the Society of Women Engineers or the Association of Women Mathematicians might also be

encouraged to expand their activities to include more direct involvement with schools and school systems in career education efforts. Perhaps federal and private monies might be provided for furthering their efforts to recruit students and disseminate positive images of career opportunities for women.

There is one additional barrier to women's full development in mathematics, particularly careers in this area, that cannot be directly removed by educational program efforts. There still exists ambivalence and conflict about appropriate roles for women. The combining of family life and careers is still more problematic for females than males. Career education programs and career counseling for women must somehow deal with the realities of this problem. The world of work outside the family is accessible to women but may carry some extra burdens or costs that are not present for men. As long as this situation exists, some females may remain aloof to attempts to increase their interests in careers and thus their achievements in mathematics.

Time and systematic efforts to educate society as a whole may eventually bring about some desired changes. Closer federal scrutiny of sexist hiring and promotion practices, the provision of better jobsite day care facilities for children of working mothers, wider acceptance of the contribution of women to the labor force, better health insurance provisions for working mothers, and other labor and health measures may in combination with better educational opportunities lead to changes in society's acceptance of women's potential for contributions to all areas of endeavor.

The foregoing suggestions for educational values and practices do not begin to exhaust the possible strategies that might be employed to increase women's participation in mathematics courses and careers and thus hopefully reduce sex differences. In summary, there appear to be five important goals related to education that could be affected directly by program efforts at the national level. First, greater awareness among females of the value and usefulness of mathematical competency for adult life, particularly in careers. This can be approached through the development of career programs, curriculum development, and public education. Second, a reduction of harmful stereotyping of mathematics as a male domain and the feminine role as incompatible with achievement and competency in mathematics. This goal can be approached through a variety of means. Third, the fostering of greater awareness and sensitivity on the part of educators, parents, and the general public of their role in promoting the full development of all talent areas in young children and the value of the first two goals. Fourth, the promotion of a truly flexible and democratic educational system that can deal with individuals and not stereotypes. Fifth, the support of educational research on the causes and correlates of sex differences in

achievement in mathematics and the development of intervention and remediation strategies including career education efforts. Elaboration of research needs are in the following section.

## Recommendations for Research

Until very recently there has been little research on factors inhibiting the intellectual and career development of women. Two types of research efforts seem indicated. First, more basic research is needed to understand the nature, extent, and causes of sex differences in mathematics achievement on tests, in course-taking, and in career choices and success. Second, it is not too soon to conduct experimental and quasi-experimental studies of intervention and remediation programs for students, educators, parents, and combinations of the three groups.

The design and implementation of both types of studies should include two considerations. There is a real need for longitudinal studies. Interpretation of future studies would also be greatly enhanced by the observance of some uniform standards for measurement and evaluation.

Too few existing studies of sex differences have been developmental and longitudinal developmental studies of child-rearing practices have rarely collected the types of information necessary to understand the dynamics of sex differences in mathematical competence and self-confidence. Some on-going longitudinal studies might be modified to include the collection of such data as parental sex-typing of activities and expectations for sons and daughters in careers and mathematics achievement. There is clearly much we do not know about the development of competence in mathematics, career aspirations and interests, values, and sex-typed or androgynous personalities. Longitudinal studies that assess a multitude of variables are needed to provide meaningful answers to the many questions that exist. A single national study of the problem may be too difficult and costly to tackle. A comprehensive plan might be developed, however, and various parts of the research divided among many different research groups.

The need for a complex but congruent research design is also true of studies of intervention and remediation programs. Lantz, et al. (1976), for example, found it extremely difficult to evaluate and compare intervention strategies funded by the National Science Foundation which varied in criteria for selection of subjects and measurement of outcomes. The interpretation of findings would be considerably enhanced by the application of uniform selection and outcome measures across studies. For example, studies that attempt

career education or counseling in different ways could be compared more readily if they used the same pre- and post-test measures.

Let us consider the following questions: At what age should career education programs begin? Are female role-models essential to career education programs? Is it more efficient to have programs for teachers or parents or counselors than programs for students? If numerous separate proposals are sought and funded which attack these questions as separate entities and employ different populations and evaluation measures there may be only noncomparable fragments of the puzzle at the end. If, on the other hand, one large research design is developed and appropriate sampling techniques employed as well as uniform pre- and post-measures, the results may prove most enlightening. We could have X number of treatment models, some designed for students, with and without role-model components, some designed for teachers, some for parents, some for counselors, and perhaps some which have components for more than one group. Each of the X number of models could be tried for Y number of age groups. Social class and mathematical aptitude of students could be controlled for by various methods. Pre- and post-test measures could be designed to encompass the goals of all the programs. Outcome measures of increased course-taking at a given age, career interests, knowledge of career opportunities, etc. could all be assessed for each sample of students to determine the relative effectiveness of each strategy. If sample populations are large, other variables such as value orientation, etc. could also be assessed. If all data were collected at uniform times and recorded in the same format, a single computer analysis could be programmed and conducted. Perhaps one institution could design and perform all the analysis while a large number of different research groups conducted the actual program aspects. Such an approach might indeed revolutionize and revitalize the educational research community. Such an approach might also be employed on longitudinal developmental studies and thus alleviate the burden placed on one research group or one set of respondents. Thus five or ten or even hundreds of small manageable longitudinal studies could be conducted simultaneously on matched or random samples. Such an approach would make it possible to assess the full range of possible correlates without studying each factor in every sample.

Let us consider some of the research questions that might be addressed by non-intervention studies, particularly those of a longitudinal or case study nature. Studies of the development of career interests and aspirations, values, and specific interest in course-taking, and achievement in mathematics should be funded. Ideally these studies should collect information on parental child-rearing practices or styles, expectations for the child in mathematics and careers, and degree of parental sex-typing of mathematics as masculine and concern for sex-appropriate behavior of offspring. Within such a study or as a separate study, data should be collected

to determine the onset of sex differences in the perception of the usefulness of mathematics and the relationship of this to other variables such as the child and parent view of mathematics as a male domain, the child's primary sex role identification, the child's apparent mathematical aptitude, and so forth. Although it would be a mammoth task to study all these variables, a single research design could be developed and somewhat different pieces of the puzzle parceled out to different investigators for samples matched on relevant variables such as social class and working status of mother. (It is difficult to know what variables should be controlled; for example, the number and sex of siblings may also be important.)

Such a design would encompass many specific testable hypotheses while also allowing for more complex analysis of the relative importance of the various factors. From this research we might learn how potent are the effects of role models in the home, the importance of parental expectations versus child-rearing practices, the home factors that lead to the development of sex-typed rather than androgynous views of self and the world, and so on.

In time we might be able to develop prediction equations for career interests and achievement. Thus, we could know which girls have the greatest likelihood of success in mathematics without special intervention efforts and which girls need the most encouragement from sources outside the home.

It would also seem desirable to approach the study of the development of math-anxiety and low self-confidence in such a way. Some preliminary research such as case studies of identified cases is needed, however, before such an ambitious project can be launched. The relationship of spatial visualization abilities to mathematics achievement and math-anxiety should also be attacked.

Some additional non-intervention studies which would not need to be longitudinal are also indicated. In some cases re-analysis of existing data banks might provide some answers. For example, we need to know more about the relationship of social-class and ethnic or racial identity to career interests, the perceptions of the usefulness of mathematics, course-taking and achievement, and so forth. We also need to understand more about school and school systems' administrative practices that foster female course-taking and achievement in mathematics.

Some additional studies of the effect of differential course-taking upon sex differences on achievement and aptitude tests might also be done in ways as to control for the effects of attitudes or even sex-typing of mathematics, and/or androgynous rather than sex-typed views of self and the world. For example, do female high school or college students who have an androgynous view of self and/or a neutral rather than masculine view of mathematics score as well as their male cohorts on tests such as the SAT-M or GRE-Q when course-taking is controlled? Another related question would be, do females who have a strong feminine

identity and a view of mathematics as a male domain take fewer advanced courses and score lower on achievement tests than would be predicted for females of similar aptitude but different perceptions of self and mathematics? Basic research on the whole issue of psychological gender identification is also needed.

Studies of the impact of peer pressure on course-taking and achievement would also be interesting. Consider these questions: Do girls who are very talented in mathematics who have friends who have strong social values and a strong perception of the importance of being feminine avoid the full development of their talents more than girls who have friends who have less sex-stereotyped values and views? Are mathematically gifted girls who pursue the development of their talents less sensitive to peer pressure; are they socially more like loners than other girls? Do adolescent girls form peer groups on the basis of their acceptance or rejection of the stereotyped feminine role?

All of the above questions are interesting to researchers. They do not exhaust the rich source of hypotheses to be studies for the advancement of the science of individual differences. They are not all equally valuable, however, for the purposes of the design of intervention programs and the development of educational policy. Perhaps some of these questions could be studied within the framework of research that simultaneously study intervention strategies.

In planning and implementing intervention and/or remediation programs, we are confronted with the question of goals. There are at least four different goals we might adopt or some combinations of them. First, we might aim at the reduction or elimination of sex differences on test performance. Second, we might attempt to reduce sex differences in course-taking at the secondary and post-secondary levels. Third, we might wish to increase women's interest and participation in a variety of career areas that require mathematical competence. Fourth, we might wish to reduce or eliminate sex-role stereotyping, particularly with respect to mathematics. This fourth goal also implies the reduction of conflict between femininity and achievement in mathematics and thus the increase in females' self-confidence in selves as learners of mathematics.

Of the four goals, the first three lend themselves most directly to attack and evaluation. The first and second goals overlap as do the second and third, while the fourth goal overlaps the first three. Therefore, in the ensuing discussion of research on intervention we will assume that all four goals are desirable.

Intervention efforts can be roughly classified in four types of strategies although a particular program might use more than one strategy. The first strategy is to create programs directly for students that aim to alter their attitudes and behaviors with respect to mathematics and careers. The second is to influence the attitudes and behaviors of significant others in the students' lives at home and school.

The third is to manipulate instructional and administrative educational practices. The fourth is to try to change the image of mathematics as a male domain by manipulating the language and messages of media, textbooks, and tests and perhaps increasing the visability of positive female role-models.

The first type of strategy would include career awareness and education programs and career counseling efforts. This could also include counseling programs for math-anxiety and accelerative programs for gifted students. The specifics of the programs might be quite varied. The literature suggests, however, that exposure to female role-models should be incorporated in the design. The exact nature of the program may be far less important than the interaction of the age of the subject and the duration and intensity of the program with the nature of the program. For example, a short-term intensive algebra program for gifted seventh-grade girls appeared to have an effect that lasted for about three years but may fade without further intervention. Some of the dependent measures that might be assessed in evaluating the impact of career education, counseling and other intervention programs are the perceptions of the usefulness of mathematics, the perception of mathematics as a male domain, course-taking, and expressed career interests and aspirations.

It might be worthwhile to consider the development of a video-tape career education program with workbook and teacher guide that could be widely disseminated. Existing commercial products of this type should be evaluated.

Ideally, career education units should be integrated with the regular curriculum so that the relationship of the skills required for a particular job could be related to the skills being learned in classes. Careers in many business areas can require a wide range of mathematical skills from basic percent and decimals to calculus and computers. Students in general mathematics courses as well as calculus might simultaneously study applications of mathematics to careers in accounting, tax-law, marketing, and so forth.

Programs that are developed for and offered to different populations should not be compared with one another. At least two levels of the population should be targeted. First, we should develop and evaluate programs for increasing women's participation in careers in mathematics and science at the professional level that are piloted on high ability students. Second, we should develop and evaluate programs for increasing women's participation in career areas that require competence in mathematics but not the more advanced levels of abstract mathematics. Programs for the student who has high mathematical aptitude should be different from programs for the student of average ability. Internship programs that place students in situations with college professors, graduate students, and research scientists, for example, are most likely to impact students who have the aptitude and necessary skills to benefit from such exposure. College professors, lawyers, and doctors are not realistic role-models for all students. A program designed to expose students to professional career possibilities in mathematics and science will be far more effective if it is directed toward the top 10 percent of the female populations with respect to mathematical aptitude than

if it is aimed at those who score below the average. Programs that ignore this factor are not likely to be effective.

The same caution applies to career counseling efforts. In our zest to increase women's participation in mathematics and the sciences, we must not overlook real individual differences and limitations. A student who scores high on a scale of mathematical or computational interest in a vocational preference inventory but has average scores on measures of mathematical and verbal aptitude should not be told to consider careers such as a professor of mathematics or an electrical engineer.

Initial research efforts aimed at improving career counseling for women might be aimed at studying the effects of non-sexist counseling. Research on the non-sexist use of existing vocational interest inventories and the development of better instruments for women should be encouraged.

The second intervention strategy would be to attempt to influence the attitudes and behaviors of significant others in the child's life. For educators, in-service courses could be developed. They might take a variety of forms. It may be more difficult to have programs for parents unless they are integrated with programs for students. In other words, it may be easier to get parents to attend meetings to explain a "new" program for students than to get parents to attend general sessions on the negative effects of sex-role stereotyping or on non-sexist child-rearing, etc.

The impact of programs for educators or parents could be compared with that of those for students by selecting teachers and parents of samples of students matched in relevant variables to those who receive more direct intervention programs. One methodological problem will be that parents and teachers of students involved in a direct program may become affected by the program in indirect ways. Ideally, we should compare the effects of programs for students with those for students and their teachers and parents and with those for teachers or parents only. In the latter cases the dependent variables could be the same as for the direct intervention studies with some additions. We would need to know the pre- and post-attitudes and behaviors of the teachers and parents. This latter type of research could become more complicated. We would need to learn whether or not the teacher program affected their attitudes and behaviors and then if this in turn led to changes in course-taking, etc., of the girls.

From a cost-effective viewpoint programs for educators would be the easiest and least expensive to undertake. Current research, alas, does not suggest that changing teacher, counselor, or school administrator attitudes and behaviors would be a highly potent program in itself. A true experimental test of this hypothesis seems crucial in terms of long-range planning and program costs.

It is difficult to envision a research design to impact peer attitudes that would not be encompassed under the general provision of career education programs. The peer group at times is an elusive concept.

A possible research issue is the determination of the "critical number" of girls required in an advanced or accelerated mathematics class to ensure the success of the girls involved.

The third strategy is to focus on research on changes in instructional modes, curriculum, and administrative practices. We might compare the effects of programs that require more mathematics courses at the high school or college level with those that maintain an optional policy. More systematic study of the effects of accelerative and non-accelerative programs for the gifted with those of programs that do not provide for the identification and facilitation of the academically able student seems warranted. A study of grouping by values and interests along with appropriate curriculum modifications would also be interesting. Do students who have social and aesthetic values benefit more from mathematics courses that emphasize the application of mathematics to the arts and social service programs than they do from courses that do not emphasize the applications of mathematics? Are self-paced mathematics classes less effective for girls who have social interests and values than for girls who are more theoretical in orientation?

Research efforts on the interaction of verbal and spatial abilities of students with instructional strategies and curricula are also needed. Perhaps students who have poor spatial abilities and high verbal abilities learn mathematical concepts in a different way from those who are less verbal and more spatial in orientation.

For the fourth strategy we must consider the ways in which society communicates sex-role stereotypes in relationship to mathematics. It is possible that girls would be more interested in learning mathematics if textbooks were less sexist. Performance on tests might be improved if the wording and context of tests were less sexist. Experimental research on the effects of textbooks and test items would seem desirable. It is difficult to envision an experiment or quasi-experiment that could assess the influence of sexism in general media.

In order to research the four strategies at minimum cost, it might be possible to capitalize upon naturally occurring experiments so that the cost of the research would not have to include the cost of treatment. For example, schools or school systems that have existing career awareness programs or mathematics programs for the gifted could be compared with schools or systems that do not offer such programs on measures of course-taking and achievement. The control schools or systems could be matched with the "treatment" schools on relevant variables such as size, available course-taking, and socio-economic levels of students. Base-rate figures for the treatment school system prior to the implementation of their special programs, if available, could also be compared with current rates of course-taking and achievement among females.

Research efforts that include the actual development and conduct of treatments should probably focus on strong rather than weak treatments. Short-term interventions such as a one-day career education program with exposure to female role-models that aim at hundreds of girls are probably far less effective than more intensive efforts that work with smaller groups of students over longer periods of time. This hypothesis can be readily tested. A somewhat more complicated question is whether or not

small but stable changes in teacher behaviors and school policies and practices will eventually impact a larger number of students over time than intensive programs for a specific sample of girls that is too complicated or expensive to ever become integrated into the basic school curriculum. Some technical questions need to be answered. For example, are live role-models significantly more effective than video-taped programs or written materials that include many female role-models? For the short-run it would be easier to use live models than to create films and video-tapes. The films or tapes, however, could be used year after year. It is probably more difficult to continually recruit live models on a yearly basis. It may be less costly to conduct an in-service course for teachers than to buy new textbooks and tests. One in-service experience for a teacher may not have, however, as lasting an effect as the continued use of non-sexist curricula materials and tests.

Thus, initial efforts to fund research should be expansive and not assume too quickly that one approach is more promising. Existing research simply does not allow us to make these decisions. It is too soon to say which type of intervention strategy will have the greatest impact. Although it will not be possible to fund every conceivable variation and combination of intervention strategies, it would seem desirable to select projects for funding that provide some balance among the types of strategies, the characteristics of the target populations, the costs, scopes, and durations of the projects.

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II

# INFLUENCES OF SELECTED COGNITIVE, AFFECTIVE. AND EDUCATIONAL VARIABLES ON SEX-RELATED DIFFERENCES IN MATHEMATICS LEARNING AND STUDYING

By

Elizabeth Fennema*

Department of Curriculum and Instruction
School of Education
University of Wisconsin

Sex-related differences in mathematics learning is a topic that has received a great deal of attention in the lay press as well as in professional, educational and psychological literature for a number of years. However, much of what is considered as "truth" is based on myth, inadequate reporting, or research studies which failed to control one important variable, i.e., the number of years of mathematics studies while enrolled in high school. One must understand as clearly as possible if, when and where, sex-related differences exist in performance in the broad intellectual area called mathematics, before one can begin to hypothesize concerning related variables. Therefore, the first major goal of this paper is to identify what is believed about sex-related differences in overall mathematics learning. In relation to this, questions will be raised about the validity of these beliefs, and one specific area will be identified as the most salient cause of sex-related differences in mathematics learning.

The second major goal of this paper is to identify cognitive, affective and educational variables which have been shown, or have been hypothesized, to contribute to sex-related differences in mathematics learning and studying. Two major bodies of professional literature will serve as the sources, i.e., the well

* Prepared with the assistance of Mary Ann Konsin.

documented mathematics education literature and the rapidly growing psychological literature related to the intellectual development of women. These two sets of knowledge all too often have been disjoint and it is necessary to synthesize relevant information from both sources. The importance of doing this is evident. Becasue of the nature of the subject matter, mathematics learning does not take place, other than incidentally, outside of formal schooling. To ignore what is known about why and how people learn mathematics as reported in mathematics education literature would indeed be foolish. However, much of what is known about how people learn mathematics must be reevaluated in light of the new insights being generated by psychologists specializing in the study of women. These psychologist are reinterpreting old data and gathering new data as questions of integrity are being asked about women and their intellectual development. In some cases the synthesis of these two bodies of literature will be relatively easy, and clear-cut conclusions, testable hypotheses or plans for action will emerge. In other cases, conflicting evidence will be apparent and new data will need to be gathered before any plans for action can emerge.

## Sex-Related Differences In The Learning of Mathematics

### Achievement in Mathematics

For the purposes of this paper, learning and achievement in mathematics are used interchangeably. Both are indicated by a performance which results from the intereaction of inherent and environmental variables. Such performance is usually indicated by scores on tests involving a variety of mathematics at cognitive levels ranging from low level computational skills to high level problem solving skills. Care must be exercised in inferring from such scores to anything about inherent ability or environmental influences. Performance on a mathematics test is merely a response to a particular set of items at a specific point in time. Such responses are usually grouped together to describe a particular set of people by measures of central tendency and deviation. At a different point in time individuals within that group might respond somewhat differently but it is assumed that the group response would be somewhat similar. Nothing can be inferred, however, about

why that group performed in that particular way. Keeping in mind this limitation, information about mathematical performance is useful. It is an indicator of how much learning has taken place and also is an efficient predictor of future success in mathematics.

Mathematics educators have used sex as a variable in research concerned with mathematics achievement for a number of years. Many general summaries have been published and most include something about comparative learning of mathematics by females and males. There is a remarkable degree of consensus about sex-related differences in those reviews published before 1974. E.g., "The evidence would suggest to the teacher that boys will achieve higher than girls on tests dealing with mathematical reasoning" (Glennon and Callahan, 1968, p. 50). "From junior high school and beyond. . . boys now surpass girls in studies involving science and mathematics" (Suydam & Riedesel, 1969, p. 129). "Sex differences in mathematical abilities are, of course, present at the kindergarten level and undoubtedly earlier" (Aiken, 1971, p. 203).

Basically all these reviews concluded that while there might not be a sex-related difference in young children, male superiority was evident by the time learners reached upper elementary or junior high school. In addition, males were definitely superior in higher level cognitive tasks which assume increasing importance as one progresses to advanced mathematical study, i.e., Algebra, Geometry and beyond. In none of these reviews was the magnitude or the educational significance of the difference discussed and no concern for explanatory reasons or remedial measures was evident (Fennema, 1974).

The reviews of sex-related differences in mathematics published since 1974, do not always draw the conclusion of male superiority in mathematics. Fennema (1974) reviewed 36 studies and concluded that there were no sex-related differences in elementary school children's mathematics achievement. She also found little evidence that such differences exist in high school learners. However, there was a trend that males excelled in higher level cognitive tasks and females in lower level congitive tasks. Fennema further concluded that more research is needed on this issue since previous research has raised more questions than it

had answered. Callahan and Glennon (1975) agree with this conclusion. Maccoby and Jacklin (1974) in a highly quoted review disagree. They state that one 'sex difference that /is/ fairly well established. . . is that boys excel in mathematical ability" (p. 351-352).

In order to clarify the reality of sex-related differences in mathematics, four major studies of sex-related differences in mathematics achievement must be specifically noted: Project Talent, the National Longitudinal Study of Mathematical Abilities (NLSMA), the National Assessment of Educational Progress (NAEP), and the Fennema-Sherman Study. The first three of these studies have had much impact on current belief about sex-related differences and as much should be examined carefully. The fourth study, results from which are in press, is the most current information available on sex-related differences.

Data for Project Talent were gathered about 1960 (Flanagan et al., 1964). This study assessed mathematics achievement (among many other things) of a random sampling of high school students in the United States (n≈440,000). Mean scores for both sexes were very low. In Grade 9 sex-related differences were negligible but by 12th grade males appear to do better. The mean difference at grade 12, while statistically significant, appears to have little educational significance (approximately one item). No attempt was made to control the number of mathematics courses subjects had taken previously. Higher percentages of males than females were enrolled in college preparatory courses so males undoubtedly had taken more math courses. Therefore, a population of males with more mathematical background was probably being compared with a population of females with less mathematical background.

In 1975 a small follow up study to Project Talent was done by testing approximately 1800 students in Grades 9-11 in seventeen of the original schools (1976). After careful statistical checks on reliability of the comparisons and adjustments for any change in school SES the following conclusions were made: (1) While the mathematics test scores were fairly stable from 1960 to 1975, the difference between female and male scores had been reduced with males scoring .4 of an item higher in 1975. (2) On computation tasks male scores had declined 17% and female scores 11% with the female means score being 8.2 points higher than the male mean score. (3) Quantitative reasoning scores

declined 8% for each sex with females scoring .6 of an item lower in 1975. It is difficult after carefully examining this data from Project Talent to conclude that males mathematics achievement was higher than that of females in 1960 or 1975.

Support for the belief that females do not achieve as well as males in mathematics could come from the NLSMA data which were gathered during 1962-67. In these multitudinous studies, sex was used as a control variable. Analyses were done independently by sex whenever significant sex by any other variable interaction was found. Unfortunately, the results from these studies have been inadequately reported and interpreted making the knowledge they could contribute to the area under consideration largely unavailable. However, a summary statement says: "Differences favoring girls were for variables at the comprehension level (the lowest cognitive level tested) and the differences favoring the boys were for variables at the application and analysis level" (Wilson, 1972, p. 95). The directors of this federally financed program abrogated their responsibility to females when they followed the above remarks with this statement: "Interpretation and comment on this pattern will be left to persons involved in the women's liberation movement" (Wilson, 1972, p. 95). The number of mathematics courses which had been taken previously by the subjects in the NLSMA studies was controlled so the conclusion reached undoubtedly was statistically valid in 1967. What is unknown is the size of the difference between the mean female and male performance scores and the educational significance of that difference.

Results from the 1972-73 mathematics data collection of NAEP have received much publicity and one sentence has been widely quoted: "In the mathematics assessment, the advantage displayed by males, particularly at the older ages, can only be described as overwhelming" (Mullis, 1975, p. 7). Inspection of this data confirms that males did outperform females at ages 17 and 26-35. However, at ages 9 and 13 differences were minimal and sometimes in favor of females. The problem of comparable populations is a concern here also. The population was selected by sophisticated random sampling techniques with no control for educational or mathematical background. Since males have traditionally

studied mathematics more years than have females, once again a population of males with more background in mathematics was being compared with a population of females with less background in mathematics. At ages 9 and 13 when the educational and mathematical background was similar, the achievement of both sexes was also similar.

The Fennema-Sherman Study, data for which were collected in 1975-76, investigated mathematics achievement in Grades 6-12 (Fennema and Sherman 1977; Sherman and Fennema, 1977; Fennema and Sherman, Note 1.) This National Science Foundation sponsored study investigated a variety of levels of mathematical learning as well as cognitive and affective variables hypothesized to be related to differential mathematics achievement by females and males. The results of this study have wide generalizability because of the diverse carefully selected sample. In Grades 9-12 ($\underline{n}$ = 1233) with subjects whose mathematics backgrounds were carefully controlled, significant differences in achievement in favor of males (approximately two items) were found in two of four schools. In Grades 6-8 ($\underline{n}$ = 1330) significant differences were found in favor of females in a low level mathematical cognitive task in one of four tested school areas. In another of the four school areas significant idfferences were found in favor of males in a high level mathematical cognitive task.

## Problem Solving in Mathematics

The discussion to this point has considered mathematical achievement as if it were a precisely defined skill or attribute. This is not so. The measures of achievement utilized, with the exception of those used in NLSMA, have been a mixture of several types of mathematical performance. There is, however, one body of literature that deals with a rather specific type of mathematics performance which is important in the understanding of sex-related differences in mathematics. That literature focuses on the process of mathematical problem solving.

The problem-solving process is important at any mathematical level and assumes increasing importance as one moves into advanced mathematics. In contrast to performing rote, computational exercises one must be able to abstract relevant data, translate

the problem into mathematical terms, decide what mathematical process(es) will solve the problem and perform these processes accurately. Schonberger (1976) thoroughly reviewed many problem solving studies which used sex as a variable. She concluded that while sex-related differences in problem solving abilities may be small, male superiority does appear to exist starting at early adolescence and increasing until maturity. Schonberger's own study did not support these conclusions, as she found no sex-related differences in mathematics problem solving in seventh grade subjects.

What, then, can be concluded about sex-related differences in mathematics learning in 1976?

1. There are no sex-related differences evident in elementary school years. This is at all cognitive levels from computation to problem solving. This conclusion has been accepted for a number of years.

2. After elementary school years, differences do not always appear.

3. Starting at about the 7th grade, if differences appear, they tend to be in the males' favor, particularly on tasks involving higher level cognitive skills.

4. There is some evidence that sex-related differences in mathematics learning in high school may not be as large in 1976 as they were in previous years.

5. Conclusions reached about male superiority have often been gathered from old studies or from studies in which the number of mathematics courses taken was not controlled. Therefore, a better mathematically educated group of males was being compared to a group of females who had participated in less mathematics education. In reality, what was being compared were not females and males but students who had studied mathematics 1-3 years in high school with students who had studied mathematics 2-4 years in high school.

Sex-Related Differences in the Studying of Mathematics

There are sex-related differences in the studying of mathematics. This is indicated by females choosing not to enroll in mathematics courses in high school and by the paucity of females in university mathematics courses. Undoubtedly, the most serious problem facing those concerned with equity in mathematical education for the sexes is ensuring that females continue their study in mathematics. In support of this statement consider some data from Wisconsin. During the 1975-76 academic year, while approximately the same number of females and males were enrolled in Algebra, in the advanced courses many more males were enrolled (see Table 1).

Table 1

Number of Males and Females Enrolled in Wisconsin in Mathematics Courses[a]

| Course | Males | Females |
|---|---|---|
| Algebra[b] | 41,404 | 41,579 |
| Geometry | 20,937 | 20,280 |
| Algebra II | 11,581 | 9,947 |
| Pre-Calculus | 3,234 | 1,917 |
| Trigonometry | 4,004 | 2,737 |
| Analytic Geometry | 1,752 | 970 |
| Probability/Statistics | 1,113 | 581 |
| Computer Mathematics | 3,396 | 1,481 |
| Calculus | 611 | 262 |

[a]Data obrained from Wisconsin Department of Public Instruction Enrollment Statistics, 1975-76.

[b]Students enrolled in one year course and two-year course.

Although only symptomatic of the effects of many variables, electing not to study mathematics in high school beyond minimal or college requirements is the cause of many females' nonparticipation in mathematics related occupations. The one variable which can be positively identified as causing sex-related differences in mathematics learning is the differential number of years females and males spend formally studying and using mathematics. Such a simplistic explanation of such an important problem seems too good to be true. However, this author believes strongly that if the amount of time spent learning mathematics is equated for females and males, educationally significant sex-related differences in mathematics performance will disappear.

A number of researchers are addressing the issue of equality of educational opportunity in relation to quantity of time spent in school. Harnischfeger and Wiley (1975) present the clearest explication of a model which shows the relation between pupil activities in school and academic achievement. Harnischfeger and Wiley believe that there is a strong relationship between academic achievement and hours of instruction. They state:

> In a school-level regression analysis, we estimated the coefficents linking verbal, reading, and mathematics achievement to number of hours of instruction per year with adjustment for pupil's prior characteristics. In terms of typical gains in achievement over a year's period we concluded that in schools where students receive 24 percent more schooling, they will increase their average gain in reading comprehension by two-thirds and their gains in mathematics and verbal skills by more than one-third. These tremendous effects indicate that the amount of schooling a child receives is a highly relevant factor for his achievement (Wiley and Harnischfeger, 1974, p. 9).

While the effect of nonelection of mathematics courses is the most important cause of sex-related differences in the learning of mathematics, explicit information concerning the causes of females electing not to study mathematics is largely unavailable. Answers to the following questions are essential.

1) What is the scope of the problem nationwide? Are there geographical, socioeconomic, or race differences in the election of mathematics courses?

2) Are there identifiable geographical areas or schools where differences exist in the election of mathematics courses by females? What are the characteristics of areas and schools where either an unusually high or low percentage of females elect to study mathematics beyond minimal requirements? Casserly (1975) offers some information regarding this question.

3) Are there sex-related differences in the type of mathematics courses being taken by the sexes? Are females taking more terminal or applied courses such as "consumer" mathematics while males take courses that lead to more advanced study?

How to change females' election of mathematics courses is unknown. In fact, changing any behavior in a systematic way appears to be an elusive, if not impossible occurrence. It has been simplistically assumed that the first step in changing behavior is to identify the causative factors of that behavior. Since the relationship between a factor and a behavior can not usually be directly observed, and in spite of the cautionary remarks by statisticians, causative factors are usually identified as those which occur in juxtaposition with the behavior under consideration. After these so-called causative factors have been identified, many believe that one has only to modify them and the behavior automatically changes in the desired way. Although this chain of inferences has some validity, there are some problems with it. Not all factors related to performance can be modified. For example, there are many studies which indicate that mothers' educational attainment is related to daughters' educational aspirations. It is obvious that while this piece of information is interesting, mothers' educational attainments are almost impossible to change.

Human behavior can not be partitioned into specific factors for other than theoretical purposes. Even when modification in an identified factor is possible, the factors exist in such a complex network of intersecting factors that modification of one is ineffective in producing long range behavioral change. Some related factors, while perhaps amenable to change, either cannot be changed by schools or the schools' possible impact is limited. For example, some data indicate that fathers have

a direct influence on girls' performance in the cognitive areas. The possibility of schools changing fathers' treatment of girls is either impossible or so prohibitively expensive in time and resources to render it impossible.

It is only sensible to concentrate the efforts of NIE in areas in which change may be effected. Therefore, this paper will concentrate on variables which are related to females' performance in mathematics and which are most amenable to change by intervention techniques within the framework of formal education. Since the concern here is with ensuring equity in mathematical education of females and males, emphasis will be upon identifying those areas related to mathematics performance in which sex-related differences have been found.

## Cognitive Variables

"Mathematics is essentially cognitive in nature; and the principle, distinguishing goals or objectives of mathematics instruction are (and should be) cognitive ones" (Weaver, 1971, p. 263). Since mathematics is a cognitive endeavor, the logical place to begin to look for explanatory variables of sex-related differences in mathematics performance is in the cognitive area. It is within this area that the most important variable can be found, i.e., the amount of time spent studying mathematics. This variable has already been discussed.

In addition to total amount of time spent in studying mathematics, what one studies is also important. One learns what is practiced. If one practices computational algorithms (a low level cognitive task) one learns to compute accurately and quickly. If one practices solving problems (a high level cognitive task) one learns to solve problems. Since there appears to be sex-related differences in levels of cognitive skills in mathematics, can one assume that females tend to spend more time doing one level of cognitive task while males tend to do another level of task? This would be easy to ascertain at the high school elvel by finding out if females and males tend to take different types of mathematics courses. It is also of vital importance to ascertain if elementary school females and males spend their time in mathematics classes doing somewhat different activities.

## Spatial Visualization

One cognitive variable that may help explain sex-related differences in mathematical achievement is spatial visualization--a particular subset of spatial skills. Spatial visualization involves visual imagery of objects, movement or change in the objects themselves or change in their properties. In other words objects or their properties must be manipulated in one's "mind's eye" -- or mentally. Even though the existence of many sex-related differences is being challenged, the evidence is still persuasive that in the American culture male superiority on tasks that require spatial visualization is evident beginning during adolescence (Fennema, 1975; Maccoby & Jacklin, 1974).

The relationship between mathematics and spatial visualization is logically evident. In mathematical terms spatial visualization requires that objects be (mentally) rotated, reflected and/or translated. These are important ideas in geometry. In fact James and James (1968) in defining geometry as "the science that treats of the shape and size of things . . . the study of invariant properties of given elements under specified groups of transformation" (p. 162) are describing accurately most conditions which are met by items on spatial visualization tests.

Many mathematicians believe that all of mathematical thought involves geometrical ideas because the total discipline of mathematics can be defined as the language for describing those aspects of the world which can be stated in terms of "configurations" (Bronowski, 1947) Meserve (1973) believes that each person who makes extensive use of all areas of mathematics uses the modes of thought of geometry at every turn and that "even the most abstract geometrical thinking must retain some link, however attenuated, with spatial intuition" (p. 249). In the Russian literature, mathematics and spatial abilities are regarded as inseparable (Kabanova-Meller, 1970). Therefore, if spatial visualization items are geometrical in character and if mathematical thought involves geometrical ideas, spatial visualization and mathematics are inseparately intertwined.

Not only are spatial visualization components an integral part of the structure of mathematics, but spatial representations are being increasingly included in the teaching of mathematics. E. g., the Piagetian conservation tasks, which are becoming a part

of many pre-school programs, involve focusing on correct spatial attributes before quantity, length, and volume are conserved. Most concrete and pictorial representations of arithmetical, geometrical and algebraic ideas appear to be heavily reliant on spatial attributes. The number line, which is used extensively to represent whole numbers and operations on them, is a spatial representation. Commutativity of multiplication illustrated by turning an array 90 degrees, involves a direct spatial visualization skill. Many other examples could be cited.

Although the relationship of mathematics and spatial visualization ability appears logical, empirical data confirming a positive relationship are less clear. Many factor analytic studies have explored this relationship and several authors have reviewed the literature. Some investigators have definitely concluded that spatial skills and learning of mathematics are not related. In 1967, Very concluded: "Research on spatial ability has failed to produce any significant correlation of (the spatial factor) with any facet of mathematics performance" (p. 172). Fruchter (1954) stated that "spatial ability is unrelated to academic performance with the possible exception of a few very specialized courses such as engineering drawing" (p. 2). Smith (1964) concluded that although "there are several studies which indicate consistently that spatial ability is important in tests which are genuinely mathematical as distinct from those which involve purely mechanical or computational processes . . . the question whether the mathematical ability is dependent on the visual factor (or factors) has not been definitely answered" (p. 127, 68).

Even in the specialized mathematical area of geometry where one would expect to find the strongest relationship, empirical findings do not indicate clearly that the two are related. Lim concluded in 1962 after a thorough review of relevant literature that the evidence for a relationship between geometric ability and spatial visualization was inconsistent and unreliable. Werdelin (1961) was not willing to conclude definitely that empirical data indicate that spatial visualization ability and geometry ability were related. However, he felt that "there is strong pedagogical reason to believe in a connection between the ability to visualize and geometric ability" (p. 39).

Other authors feel that data indicate a positive relationship. In 1951, Guilford, Green and Christensen concluded that spatial visualization ability helped in solving mathematics problems. French (1951, 1955) showed that successful achievement in mathematics depends to some extent on use of spatial visualization skills. In a recent review, Aiken (1973) concluded that spatial-perceptual ability was one of the "most salient" mathematical factors extracted in various investigations. Obviously, the relationship between learning in mathematics and spatial ability is not clear and the need for more data is great.

Even less is known about the effect that differential spatial visualization has on the mathematics learning of females and males. Indication that the relationship between the learning of mathematics and spatial visualization is an important consideration, is the concurrent development of sex-related differences in favor of males in mathematics achievement and spatial visualization skills. No significant sex differences in either mathematics achievement nor spatial visualization skills have been consistently reported in subjects 4-8 years old. Sex differences in performance on spatial visualization tasks become more pronounced between upper elementary years and the last year of high school and the differences show a pronounced increase during this time span (Maccoby & Jacklin, 1974). Sex differences in mathematical achievement that do exist also appear during this time span (Fennema, 1974).

It appears reasonable, therefore, to hypothesize that since there is a concurrent developmental trend and since tests of spatial visualization contain many of the same elements contained in mathematics, the two might be related. Perhaps less adequate spatial visualization skills may partially explain sex-related differences in achievement in mathematics.

The Fennema-Sherman study specifically investigated the relationship between mathematics achievement and spatial visualization skills. These data do not support the idea that spatial visualization is helpful in explaining sex-related differences in mathematics achievement. In this study of females and males (Grades 6-12) enrolled in mathematics courses, few sex-related differences in either mathematics achievement or spatial

visualization skills were found. The two were related ($r = .5$) similarly for both sexes and spatial visualization appeared to influence both females and males equally to continue studying mathematics.

Several areas should be investigated to clarify the relationship between spatial visualization skills and females' performance in mathematics:

1) What is the impact of spatial visualization skills on females' learning of geometry and their election of additional mathematics courses?

The big dropoff in the number of females in mathematics courses occurs after 10th grade, when students are enrolled in traditional Euclidean geometry classes. Although no precise data is available, students appear to view geometry classes differently than they view other mathematics classes. This view of geometry may be a strong influence on deciding whether more mathematics courses are taken. More information is needed related to this: What influence does geometry have on the decision to elect more mathematics courses? Because of the spatial visualization component of geometry, are weaker spatial visualization skills a determining factor of success in this area? There has been some research conducted in this area but none regarded as definitive. In relation to geometry, another area needs investigation. Is the deductive component of geometry a factor in females' success or election of more mathematics courses?

2) What is the effect of spatial skills on mathematical learning at various developmental levels?

Smith in 1964 hypothesized that while spatial ability may not be realted to mathematics ability at beginning stages of mathematics learning, advanced mathematics learning increasingly depends upon spatial ability (Smith, 1964). It would appear that this hypothesis was made after surveying a number of studies which used high school or college students as subjects and relatively sophisticated mathematical ideas as criterion measures. Little or no data were presented from studies with younger learners. However, in 1964 one could have built a strong argument to support the idea that spatial ability was not related to mathematics at beginning stages of mathematics learning. Little

or no geometry was taught at the pre-high school level and most pre-high school mathematics tests focused primarily on arithmetical/computational ideas. Werdelin (1958) found computation to be negatively correlated with spatial ability. Therefore, Smith's hypothesis that spatial ability was not related to mathematical ability at beginning stages of mathematics learning was believable in 1964. The tests used to measure mathematics achievement probably included few items relevant to spatial ability and as such reflected the mathematics program of most schools previous to 1964.

Since 1964 a major change has taken place in most K-12 mathematics curricula. Geometry has become an integral part of the entire mathematics curriculum. More emphasis is being placed on the structure of mathematics and increasingly mathematics is taught as an interrelated system of ideas. In order to learn new ideas, learners are dependent upon the presence of prerequisite ideas in their cognitive structure. Little is known about the importance of spatial ability in the acquiring of these prerequisite mathematical ideas upon which all advanced mathematical knowledge is based. The influence of spatial ability on learning elementary mathematical ideas may be of the utmost importance.

Developmental psychologists patterned on Piaget have theorized that at different stages of cognitive development certain modes of thought predominate. Ideas are added to one's cognitive structure by utilizing actions, symbols which represent those actions, and symbols alone, in somewhat different blends. According to this theory, mental structures are formed by a continual process of accommodation to and assimilation of the environment. This adaptation (accommodation and assimilation) is possible because of the actions performed by the individual upon the environment. These actions change in character and progress from overt, sensory actions done almost completely outside the individual to partially internalized actions which can be done with symbols representing previous actions; to complete abstract thought done entirely with symbols. Thus, development in cognitive growth progresses from the use of physical actions to form schemas to the use of symbols to form schemas, i.e., learners change from a predominant reliance upon physical actions to a predominant reliance upon symbols.

Mathematical educators have increasingly accepted this theory of cognitive development and have translated it into educational practice by an increased emphasis upon the instructional use of three modes to represent mathematical ideas, i.e., concrete (enactive), pictorial (iconic), and symbolic. Such a belief suggests that the blend of the usage of these representational modes should reflect the cognitive developmental level of the learner. Particularly at early stages of mathematical learning it is important to provide learners with concrete representations of mathematical ideas while symbols assume increasing importance as learners mature and mathematical ideas become more complex.

Most concrete and pictorial representations of mathematical ideas include spatial attributes, some of which are relevant to the mathematical idea being taught and some of which are not. The only way to add mathematical ideas to one's cognitive structure at early developmental levels is by interaction with concrete or pictorial materials which represent those ideas, and since those representations depend heavily on spatial attributes, if one is hampered in perception of those spatial attributes then one is hampered in learning those early mathematical ideas. Without knowledge of these ideas, it is impossible to learn advanced mathematics. Therefore, spatial visualization appears to be important at early stages of learning.

Sherman (1967) has suggested that boys outperform girls on spatial tasks because they develop their spatial skills by participating voluntarily in more spatially oriented activities. Girls learn to read more easily than do boys. Because of ease of use of symbols, i.e., reading, do girls voluntarily, or are they encouraged to rely more heavily on symbols to learn mathematics rather than using concrete or pictorial representations? If so, perhaps inadequate usage of spatial representations may hamper both the development of their spatial skills and more advanced mathematical learning. No data are available to give insight into this problem. Empirical data from studies dealing with the use of various representational modes are not conclusive even about the value of concrete and pictorial representations. As far as is known no study has included spatial visualization as a factor. Certainly, more data are needed.

3) What is the interaction effect of spatial visualization ability and other abilities on achievement in mathematics?

Werdelin (1961) showed that girls proved verbal theorems better than boys but were less able to translate words into figural images and then to transform those images in a directed way. Another truism, which is still accepted by Maccoby and Jacklin (1974), is that females' verbal ability is more highly developed than is males'. Does the development of verbal ability in some way interfere with development of spatial ability? Werdelin (1958) in a factor analytic study found one spatial-visual factor in high school students which was related to a factor he called mathematical reasoning. Interestingly, he found the correlation between his spatial-visual factor and a numerical (or computational) factor was negative. Females often score higher on tests of computation than do males. Perhaps higher development of numerical or computational ability interferes with development of spatial-visualization ability. Both of these questions appear related to the earlier one of the impact of spatial ability on early mathematical learning. Does facility with symbols--computational or verbal--interfere with development of spatial skills?

4) What is the effect of possessing a greater variety of well developed abilities on mathematics learning?

Harris and Harris (1972), Werdelin (1958), and Very (1967) have shown a larger number of space factors for males than for females. Werdelin (1961) concluded that if one could attack a problem either verbally or spatially, one would be more apt to be able to solve it. His data showed that boys were superior on items which measured the ability to comprehend the organization of a visual figure and to reorganize it. Where items could be solved by verbal means and did not require that the problems be translated into a mental figure, no sex differences were found. If males have developed more abilities than have females they are able to attack mathematical problems in a variety of ways and thus score higher on mathematical achievement tests.

5) What sex differences in mathematics achievement would be found if spatial visualization were not a factor?

Tittle (1973) has shown that many tests commonly used to measure achievement are sexually biased. If a mathematics test contains many items requiring spatial skills in their solution, females possibly will not do as well as will males. It would be very interesting to construct a test that had little or no spatial content in it and compare the sexes on achievement. Perhaps no differences will be found if the test content is controlled in the spatial area. On the other hand, spatial visualization may be such an integral part of higher mathematical thinking that eliminating spatial aspects of mathematics tests too narrowly restricts the area of mathematical thinking. This aspect should be investigated.

<u>Cognitive Style</u>

Although not traditionally viewed as a cognitive variable, cognitive style utilized in the learning of mathematical content as well as in development of problem solving strategies appears to be important in understanding sex-related differences in mathematics. Cognitive styles based on two somewhat different aspects need to be considered, i.e., global-analytical and spatial-verbal. One appears not to be a useful avenue of investigation while the other does.

Witkin has been the foremost expositor of a global-analytical dichotomy in perception and a thorough review of his work can be found in Kagan and Kogan (1970). Witkin has consistently reported sex-related differences in tests measuring global-analytical skills. Since mathematics is noted for analytic thinking (e.g., Analytic Geometry), performance on tests measuring something called analytic ability has wide appeal as an explanation of less adequate female mathematical learning. However, this relationship appears to be a semantic relationship at best. Analytic in one case does not mean the same as analytic in the other. The analytic ability described by Witkin appears to involve performance on perception tasks and

the analytic ability required in mathematics involves a high level cognitive task with as yet poorly understood dimensions. Therefore, the analytic-global dichotomy suggested by Witkin does not appear to be a useful tool in understanding problem solving strategies or the learning of mathematics (Schonberger, 1976). In addition, one of the more important components of the analytic-global tasks affecting mathematics learning may be the spatial dimension included in such tasks (Sherman, 1967).

One cognitive style dimension which may help in understanding sex-related differences in mathematics is the blend of spatial-verbal skills utilized in mathematical problem solving. Many mathematical problems can be solved by the use of symbols and/or by the use of figures or drawings. For example, consider the problem shown in Figure 1.

What is the area of the figure enclosed by the heavy lines?

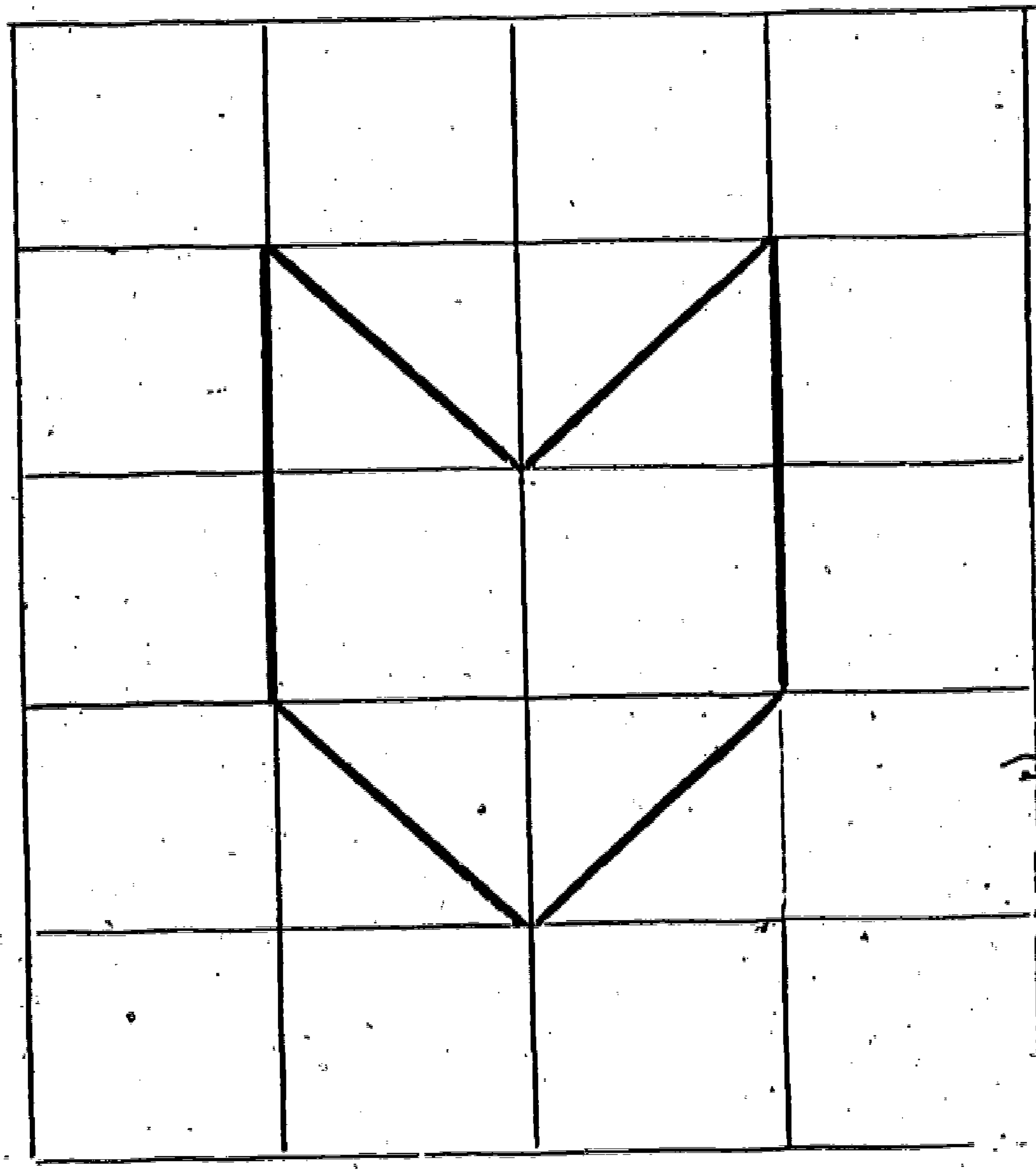

Figure 1.

This problem can be solved by counting the squares and half squares included in the figure. Thus 1 + 1 + 1/2 + 1/2 + 1/2 + 1/2 = 4 square units. The problem can also be solved by mentally moving the half squares at the top or bottom of the figure so that a 2 x 2 square results which has an area of 4 square units. In both solutions, verbal skills and spatial skills are utilized. However, the blend is somewhat different with verbal skills used predominantly in the first solution and spatial skills used predominantly in the second solution.

It has already been said that little is known about how spatial visual skills are utilized in learning mathematical ideas. Even less is know about how or if, the verbal-spatial dimension of cognitive style is utilized in solving mathematical problems. It seems reasonable that there may be sex-related differences in the use of this cognitive style. Such differences may be an important contributor to differences in mathematics studying by the sexes. This area needs investigation.

### General Intelligence and Components of Mathematical Ability

Without a doubt the major intellective factor related to mathematics learning is general intelligence as defined by any number of intelligence tests. However sex-related differences are not apparent from these tests (Maccoby & Jacklin, 1974). Such ability does not help explain sex-related differences in mathematics performance (for a thorough review of this, see Armstrong, 1975).

Another approach to understanding mathematical ability has been an attempt to identify its components through factor analytic techniques. Sex has been used as a variable in many such studies. Synthesizing these studies is difficult. Either no sex-related differences in factor structures have emerged or reported differences have not been replicated. Aiken (1971) has done a review of factor analytic studies

dealing with mathematical ability and this review reflects the knowledge base in this area. He concludes that "inter-individual and sex differences in mathematical abilities are, of course, present at the kindergarten level and undoubtedly earlier" (p. 204). However, he does not specify what these differences are or indeed cite studies which confirm that differences exist in any consistent pattern. Many of these studies appear to have been carefully designed implemented and reported, but at the present time information gained from the studies has not helped in understanding sex-related differences in mathematics. It is difficult to believe that further investigation in this area would be useful.

Verbal Variables

Verbal ability as measured by language tests of various kinds (vocabulary, spelling grammar, reading, etc.) correlates highly with mathematical performance (Aiken, 1971). However, research relating this variable and mathematics does not provide insight into sex-related differences in verbal ability favor females (Maccoby & Jacklin, 1974). Since verbal factors are so important to mathematics, why don't females achieve better in mathematics than do males? Little or no data are available to answer this intriguing question.

One important area could be considered. Perhaps the verbal factor related to mathematics is somewhat different from the verbal factor traditionally examined. In order to learn or to use mathematics one must be facile with mathematical language. Language is vital to mathematics. Starting with numbers and numeration and continuing into the basic mathematical operations of addition, subtraction, multiplication and division, mathematics has a highly specific symbolism and syntax which must be learned. As one progresses to higher mathematical levels, symbolism is increasingly used to express ideas. This symbolism is unique to mathematics; it is isolated within mathematics; and without a thorough knowledge of it, abstract mathematical thought is impossible. It would appear that while the verbal

factor in mathematics might be very similar to the verbal factor of words, there are also differences. Conciseness is one difference. One mathematical symbol is equivalent to many words resulting in a high concept density in mathematics writing. Another difference is the learning of the symbols. Learners are embedded in a social milieu which constantly bombards them with words. Not so with mathematical symbols. Their use is restricted to a great degree to mathematics classes in schools. Whether or not such a specialized verbal factor can be identified is unknown, but it appears reasonable that such a factor exists and that it might provide some understanding of sex-related differences in mathematics study.

## Affective Variables

Most educators believe that affective variables are important contributors to the learning of mathematics and the relationship between the two has been the object of many studies. Attesting to this is the availability of several excellent reviews (Aiken, 1970a; Suydam and Weaver, 1975; Callahan and Glennon, 1975; Aiken, 1976). However, conclusions reached in these reviews appear to be contradictory. Suydam and Weaver (1975) summarizing elementary school studies state: "There is no consistent body of research evidence to support the popular belief that there is a significant positive relationship between pupil attitudes toward mathematics and pupil achievement in mathematics. . . . We have little research basis for believing that these two things are causally related" (p. 1-3). Also reviewing elementary school studies for the same age Callahan and Glennon (1975) agree with Suydam and Weaver. They conclude that the state of the art "makes it difficult to present compelling research evidence . . . that positive attitudes play an important role in contributing to mathematics achievement" (p. 80). Aiken (1976) states: "When

attitude scores are used as predictors of achievement in mathematics, a low but significant positive correlation is usually found" (p. 295) at the elementary, secondary, college undergraduate and postgraduate levels.

Part of the contradictory conclusions can be explained by the age of the subjects being considered in the reviews. Two reviews (Suydam and Weaver, 1975; Callahan and Glennon, 1975) were basically concerned with children in Grades 1-6. Problems of assessing attitude in these grades have not been addressed adequately and lack of carefully designed measuring instruments may have caused reviewers to seriously question any significant differences reported. Aiken, in his review, was concerned with a much broader age spectrum. Even while recognizing the serious problems connected with the studies of young children, he was willing to accept the evidence as having some validity because the results coincided with studies having older subjects.

The most serious problem presented by the work done in this area is the definition used of the construct "attitude". One flaw is the impreciseness of the definition. For example, Callahan and Glennon (1975) define an attitude as children's liking or disliking of mathematics. Aiken in one place (1970a) offers a definition basically in agreement with Callahan and Glennon and in another (Aiken, 1970b) defined attitudes as "variables which are not explicitly measured by tests of ability" (p.28). (One wonders if the color of hair of the subjects might be an attitude.) What is the actual dimension that has been measured in studies dealing with attitudes? Does it involve euphoria, depression, confidence in performing the task, belief that mathematics is useful, recognition of success from peers, anxiety, or sense of failure? These dimensions are quite often included in a single scale which purports to measure "attitude".

Another serious flaw in many attitude studies is the global definition of mathematics used. Mathematics is a complex discipline involving many kinds of related

but diverse subject matters and skills. To assume that a person feels the same toward each part of mathematics is not reasonable. For example, computing the answers to 50 three digit by three digit multiplication problems could easily arouse different feelings in a person from those aroused when solving a mathematical puzzle.

Taking into consideration the problems not addressed by the research reported to date, several tentative conclusions can be reached.

1) There is a positive relationship between attitude and achievement which seems to increase as learners progress in school.

2) Attitudes toward mathematics are fairly stable--particularly after about the sixth grade, although one longitudinal study showed a marked decrease from 6th grade to 12th grade (Anttonen, 1969).

3) Grades 6-8 seem to be critical in the development of attitudes.

4) Extremely positive or negative attitudes appear to be better predictors of achievement than more neutral feelings.

5) There are sex-related differences in attitudes toward mathematics.

Even though there is consensus that sex-related differences in mathematics attitude exist, the magnitude and specific dimensions of these differences are unclear. Suydam and Weaver (1975) quote studies with contradictory results and say that in other studies no significant sex-related differences were found. Aiken states that the correlations between attitude and achievement generally are somewhat higher for girls than for boys and that significant differences in attitudes are frequently found to favor males over females. Basic agreement with this latter conslusion was reported in the Fennema-Sherman study with learners in Grades 6-11 (Fennema-Sherman, 1977; Sherman-Fennema, 1977; Fennema and Sherman, Note 1).

The chicken and egg question concerning attitude and achievement is relevant. Do poor attitudes result in poor achievement or vice versa? Undoubtedly there is an interaction between the two and change in one affects the other. However, there is some evidence that variables other than achievement influence attitudes toward mathematics. The influence of parents appears to be significant in the formation of attitudes. The father appears to be the dominant influence. However, a note of caution must be interjected. In 1976 fathers overall have much higher levels of mathematical training and use mathematics more. Since the studies from which the conclusion of father influence was drawn did not control mathematical knowledge of the parents, mathematical sophistication is probably the relevant variable instead of the sex of the parent.

Many people believe that teachers' attitudes toward mathematics are important determinants of pupils' attitudes. Some studies support this while others do not. Most studies have been in the elementary school where partitioning teacher effect into one subject area is risky at best. Working with middle school or high school teachers who have selected mathematics as an area of specialty also presents difficulty as long as one is only concerned with the narrow "liking-disliking" definition of attitude. These teachers, because of the similarity of their mathematical background, undoubtedly show little variance in their liking of mathematics. Overall, the data available suggest that teachers do affect pupil attitude development, both by the teacher's attitude and effectiveness. There also appears to be sex of teacher by sex of pupil interaction.

It must be stressed that all conclusions about attitudes toward mathematics are tentative and based on studies which have not defined "attitude" similarly. In addition, the measuring instruments used undoubtedly did not control for sex-related concerns which may have resulted in biased results. It appears that past research on attitudes toward mathematics is not immediately helpful in understanding or planning interventions to modify sex-related differences in mathematics achievement.

Some insight is gained when a broader definition of feelings toward mathematics is used, i.e., the idea of affective variables as defined by Krathwohl et al. (1964). Affective variables are those which deal with feelings, interests, attitudes, values, and emotional sets or biases. Of this infinite set of variables, those will be discussed which seem helpful in understanding sex-related differences in mathematics study and learning and which may be amenable to change by direct intervention procedures.

Stereotyping Mathematics as a Male Domain

It is commonly accepted that mathematics is stereotyped as an activity more appropriate for males than for females. It has been believed that the sex typing of mathematics as male starts in the elementary school, becomes stronger during adolescent years and is solidified as a male domain by adult years. However, Stein and Smithells (1969) and Stein (1971) provide evidence that mathematics is not considered masculine by females and males until adolescent years and even during these years is not ranked as highly masculine as are spatial, mechanical and athletic tasks. Bobbe (1971) found that with fourth and sixth grade subjects, arithmetic was judged to be feminine by girls while boys judged it to be appropriate for both sexes. The Fennema-Sherman study indicated that females in grades 6-12 deny that mathematics is a male domain. While the males in the study did not strongly stereotype mathematics as a male domain, at each grade they stereotyped it at significantly higher levels than did females. In the adult world, it is a fact that the use and creation of mathematics is predominantly a male domain. Stein and Smithells (1969) offered evidence that in 12th grade, females perceived this fact and were responding to the reality. The females in the Fennema-Sherman study may have been responding to this reality by their overt act of not electing to enroll in mathematics courses.

In conclusion, there is no compelling evidence at the present to conclude that prior to high school, mathematics is strongly sex typed. However, during high school years females increasingly behave as if they are aware that mathematics is for males. Throughout high school years, males tend to stereotype mathematics as a male domain at higher levels than do females. Several areas related to the sex typing of mathematics should be investigated.

1) Are certain mathematical areas more sex typed than others? E.g., is problem solving stereotyped as male and computational tasks stereotyped as female?

2. What is the effect on females of males' sex typing mathematics as masculine?

3. Is sex typing of mathematics a stable variable?

Achievement Motivation in Mathematics

Closely related to and affected by stereotyping of mathematics as a male domain is achievement motivation in mathematics. Mathematics is a difficult subject to learn and in order to achieve well in mathematics, one must be highly motivated. Unfortunately, achievement motivation as a construct, has been validated largely by using tasks with arousal conditions which correlate highly with masculine personality characteristics, such as competitiveness and leadership (Stein and Bailey, 1973; Mednick and Weissman, 1975). Because these traits have not been positively associated with the female sex role, females have tended to exhibit them to a lesser degree than have males. As a result, it has been concluded that females have less achievement motivation than do males. However, Stein and her associates (see Stein references) offer an alternative interpretation. They suggest, and offer some data to support the belief, that achievement motivation is exhibited in areas which are stereotyped as appropriate for one's sex. Therefore, according to this theory, if mathematics is

perceived as more appropriate for males, then females will have less achievement motivation in this area.

Little is known about the impact on learning mathematics of the psychological construct of achievement motivation. Much is being inferred from psychological studies utilizing subjects' responses made during short time intervals. While such studies result in testable hypotheses in a school setting, direct inference about mathematics learning is impossible. Two examples illustrate this point. 1) Stein and her colleagues have written at length on achievement motivation as it relates to sex. The studies rest on a sound theoretical base that achievement motivation is area specific and not a generalized motive. They indicate rather clearly that the sex typing of a subject is related to each sex's achievement motivation in that subject. One method used to assess level of achievement motivation is length of time a subject will pursue a task in an experimental situation. This is labeled persistency on task. Certainly persistency is related to the learning of mathematics. However, the relationship between persistency in a relatively short experimental situation and persistency on a day to day basis over a school year's time is largely unknown--and it is the latter which is important to the learning of mathematics. 2) The major relevant conclusion of the Stein studies, that of sex typing of mathematics by both adolescent girls and boys, was obtained by an experimental task involving an addition problem. There is little addition in high school mathematics. Assuming that learners respond to an addition task in high school in the same way they respond to algebra or geometry tasks is erroneous.

It is unknown how achievement motivation is related to sex-related differences in mathematics learning. Information concerning several questions would be helpful.

1) Are there sex-related differences in achievement motivation in mathematics? If these differences exist, do they appear at all grade levels?

2) How stable is achievement motivation on a day by day basis in mathematics classes and are there sex-related differences in this stability?

3) Can achievement motivation be improved in a school setting?

Poffenberger and Norton (1963) believe that large societal influences can improve males' achievement motivation. They examined grades from 1955-60, the post Sputnik era, in which there was great hue and cry about Russian superiority in space technology. Male grades showed marked improvement. Poffenberger and Norton attribute this to improved achievement motivation because of the competitiveness aroused in males. If this belief is valid, it might be that one result of the current social climate that is beginning to permit achievement by females will be females' improvement in mathematical skills.

<u>Motive to Avoid Success in Mathematics</u>

Since Horner (1968) first offered evidence that females--particularly high achieving females--fear success in male appropriate areas, much has been written about such a motive as an explanation of females' existing underachievement. In a simplistic way the theory goes something like this: In achievement areas identified as male where females would be competing with males (e.g. medicine, law, business, mathematics) females with the capacity to perform at high levels will not attempt to achieve. They refuse to compete because if they are successful, they fear a variety of sanctions from males. Theoretically the motive to avoid success becomes increasingly salient during high school and college years. Some studies have supported this theoretical construct while others have not (Mednick and Weissman, 1975). The Fennema-Sherman study did not show that high school females feared success in mathematics any more than males did nor that the motive became stronger as subjects progressed through high school.

It would be profitable to gain more information on types of mathematical activities and the motive to avoid success. Mathematical problem solving and games often involve direct competition. Such activities encourage mathematics learning and provide enjoyment. Do females exhibit a fear of success when directly competing with males? If such a motive inhibits females' participation in competitive mathematics activities, their mathematics learning might be inhibited.

## Effectance Motivation in Mathematics

One variable, which has been hypothesized to show a sex-related difference, is effectance motivation. This motive can be "inferred specifically from behavior that shows a lasting formalization and that has characteristics of exploration and experimentation" (White, 1959, p. 323). It is closely related to problem solving activity and is often called intrinsic motivation. This motivation would encourage learners to participate in mathematical activities at high cognitive levels. Some believe that females are not so involved in problem solving activities as are males (Carey, 1958; Kagan, 1964). The Fennema-Sherman study found no sex-related difference in this variable at any grade level from 6-12. However, because this type of motivation is so essential to the learning of mathematics further investigation is warranted.

## The Confidence-Anxiety Dimension in Mathematics

One tends to do those things one feels confident to do and to avoid activities which arouse anxiety. This confidence-anxiety dimension, as it relates to mathematics learning, is one of the more important affective variables that helps explain sex-related differences in mathematics learning.

The relationship of anxiety and mathematics learning has been explored by a variety of methodologies and with instruments purported to measure debilitative or facilitative anxiety in general and/or specific to

mathematics. Callahan and Glennon (1975) conclude that "anxiety and mathematics are related. In general high anxiety is associated with lower achievement in mathematics" (p. 82). Reports from NLSMA indicate that across Grades 4-10 decreases in facilitating anxiety appeared, with females' scores decreasing more than males' scores. Debilitating anxieties increased for females during these grade levels (Crosswhite, 1972).

Confidence per se has not been given specific attention as it relates to mathematics except in the Fennema-Sherman study. However, self-concept, which appears to be defined in the various scales as self confidence, has received much study. Leviton (1975) and Primavera et al. (1974) reviewed the literature dealing with self-concept and both concluded that a positive relationship exists between academic achievement and self esteem. Brookover and Thomas (1964) offer evidence that self concept is not generalizable but related to specific academic areas. Callahan and Glennon conclude that there is a positive relationship between self-esteem and achievement in mathematics. Others have also recognized the importance of academic self concept in learning mathematics (Bachman, 1970; Fink, 1962).

Although both confidence and anxiety have been defined as separate traits, it appears in relation to mathematics, they are very similar. In the Fennema-Sherman study an attempt was made to measure both confidence and anxiety. A high rating on the confidence scale correlated higher ($r$ = .89) with a low rating on the anxiety scale. While it may be possible to talk about the two independently it doesn't appear to be useful.

The literature strongly supports the fact that there are sex-related differences in the confidence-anxiety dimension. It appears reasonable to believe that lesser confidence, or greater anxiety on the part of females is an important variable which helps

explain sex-related differences in mathematics studying. Crandall et al. (1962) concluded that girls underestimate their own ability to solve mathematical problems. Others have concluded that females feel inadequate when faced with a variety of intellective, problem solving activities (Kagan, 1964). Maccoby & Jacklin (1973) reported that "girls tend to underestimate their own intellectual abilities more than boys do" (p. 41).

In the Fennema-Sherman study, at each grade level from 6-12, boys were significantly more confident in their abilities to deal with mathematics than were girls. In most instances this happened when there were no significant sex-related differences in mathematics achievement. In addition, confidence in learning mathematics and achievement were more highly correlated than any other affective variable and achievement ($\underline{r}$ = .40). Confidence was almost as highly related to achievement as were the cognitive variables of verbal ability and spatial visualization.

More information is badly needed concerning females' feelings of confidence, what contributes to its development, and possibilities of intervening to change it. At what age should this intervention take place? Currently, there are a number of mathematics anxiety clinics operating at the university level. It appears that, while changes could take place with university age students, it would be more profitable to intervene with much younger students. Mathematics learning is cumulative and if a low level of confidence is operative beginning at sixth grade, the learning that has taken place during the middle and high school years, as well as one's willingness to elect to study mathematics, will have been affected. If females could be prevented from developing feelings of anxiety or lack of confidence, more positive attitudes toward studying mathematics might result and their mathematics learning would be improved.

## Usefulness of Mathematics

A different kind of affective variable is belief in the personal usefulness of mathematics. Hilton and Berglund (1974) and the Fennema-Sherman study provided data indicating that females to a lesser degree than males believe that mathematics is personally useful. However, the difference between female and male beliefs about the usefulness of mathematics was not as great in the Fennema-Sherman study as it was in the Hilton-Berglund study. This may indicate that the beliefs of females are becoming more similar to males in this aspect.

## Other Affective Variables

There are a number of other variables which have been hypothesized to contribute to sex-related differences in mathematics achievement. Several appear to be only symptoms of the basic problem of not electing to study more mathematics. Intervention with these variables would be similar to treating the swelling of jaws when one has the mumps. Such variables include extra curricular activities in mathematics and mathematics related courses studied.

Another variable whose educational implications are unknown is locus of control. Locus of control, as it is relevant here, is the extent to which a person feels responsible for her/his success or failure in academic areas. Ordinarily studies have indicated that females feel they have less control than males do over success or failure and females are more apt to attribute their success or failure to something outside of themselves such as fate or luck (Crandall et al. 1965; Messer, 1972). In addition females expect less success than do males and this appears as early as kindergarten (Frieze, 1975). However, other studies have somewhat countered this trend and have suggested that the perceived sex appropriateness of a task influences whether one feels personal control over the task or expects success. Translating this psychological dimension into implications for education has not yet been done. If investigation is to be done in this area, such translation must receive top priority.

## Critical Educational Variables

There are sex-related differences in the final outcome of mathematical education due in large part to females' reluctance--if not refusal--to elect to study mathematics. Some intervention is essential at the present time to ensure equity in mathematical education for both sexes. However, before effective intervention can be planned, more information is needed about critical variables which are amenable to change and important in the educational process.

### Learners

The critical question of which females should be the target population of any planned intervention is not easily answered. Females with the intellectual abilities as well as emotional drive to succeed in mathematics related curricula at colleges/universities are one obvious population. It is probably within this population where the greatest success will be realized. At least two other groups of females should also serve as target population: i.e., the precocious and the non-college bound student. There is strong evidence that intervention with the precocious is extremely difficult (Fox, 1975). Perhaps the most difficult group, as well as the group which needs intervention the most, is females from poverty areas and in particular, black females. The NAEP results indicate that the lowest group of achievers on consumer type problems is black females. It is outside the scope of this paper to make decisions concerning which group of females should receive what percentage of attention. In some way this problem must be resolved humanely, rationally and efficiently.

### Teachers

Teachers are the most important educational influences on students' learning of mathematics. From kindergarten to high school, learners spend thousands of hours in direct contact with teachers. While other educational agents may have influence on educational decisions, it is the day by day contact with teachers which is the main influence of the formal educational institution. Part of the teachers' influence is in the learners' development of sex role standards. These

sex role standards include definitions of acceptable achievement in the various subject areas. It is believed that this influence by teachers is exerted through differential treatment of the sexes as well as expectations of sex-related differences in achievement.

Schonborn (1975) concluded that many studies have indicated teachers treat female and male students differently. In general, males appear to be more salient in the teachers' frame of reference. Teachers' interaction with males is greater than it is with females in both blame and praise contacts. Teachers also reinforce in both females and males sexually stereotypic behavior deemed appropriate for their sex (Sears and Feldman, 1966). Brophy and Good have been the major investigators of teacher treatment of females and males. Their well designed studies have been concerned basically with investigation of males' treatment by teachers. In addition to sex of student they often include sex of teacher as a variable. In several studies they have concluded that girls and boys receive equal treatment. However, the data from one of their major studies shows that while the sex of the teacher was unimportant, high achieving high school boys received significantly more attention in mathematics class than any other group (Good, Sikes & Brophy, 1973). Another study involving first grade reading replicated this trend at nonsignificant levels (Good and Brophy, 1971). One must question why no conclusion was reached about inequitable treatment of high achieving females.

Several unanswered questions remain:

1) What are the effects of differential teacher treatment and expectations on achievement and election of mathematics courses?

2) Do teachers differentially reinforce males and females for specific kinds of mathematical or sexually stereotypic activities? Are males being reinforced more for problem solving activities while females are reinforced for computational activities?

3) What is the effect of sex of teacher on mathematical achievement of boys and girls? While O'Brien (1975) reports no sex of teacher effect, Good, Sikes and Brophy (1973), and Shinedling and Pedersen (1976) report that male students do best in quantitative scores when taught by male teachers.

## School Organization

There is some evidence that schools do influence sexual stereotypes. Minuchin (1971) concluded that children who attended schools categorized as traditional or modern differed in their sex-typed reactions. The interaction of the sexes was different in those schools, also. In the most traditional school boys became leaders in problem solving while girls became followers. This was not so in the less traditional schools. The sex role behavior of children attending traditional schools was more rigid than children attending liberal schools.

Some schools are remarkably more effective in persuading females to attempt high achievement in mathematics. Casserly (1975) identified 13 high school which had an unusually high percentage of females in advanced placement mathematics and science classes. She concluded that the schools had identified these girls as early as fourth grade and the school teachers and peers were supportive of high achievement by the females. Rowell (1971) pursued the same type of investigation in attempting to identify schools and their characteristics which produced high achieving females in science. Studies identifying and describing those schools which are particularly successful in encouraging females to enroll in mathematics beyond minimal requirements should be done.

Many are advocating that female only clases will result in equity in mathematical education. The argument for this type of school organization goes something like this. Because peer pressure against female competitiveness is too strong a force, females will not compete against males in mixed sex classrooms. Female

leadership (in problem solving in this case) is only able to emerge when competition with males is eliminated. Teachers will not have different sex-related expectations and behaviors if only one sex is present. Single sex classrooms appear to provide a simple solution to a complex problem. However, the weight of evidence found does not support this type of grouping. Conway (1973) convincingly argues that throughout history separate education for the sexes has resulted in inferior education for females. Keeves, (1973) after a careful and thorough review of mathematics and science education in 10 countries, concluded that the "extent to which a community provides for education in single sex schools would appear to indicate the extent to which it sees its boys and girls requiring different preparation for different societal roles" (p. 52). He argues that "in so far as a community has different expectations for different groups of its members and proceeds to mould its future members through different organizations, then it fails to provide equal opportunities for individual development" (p. 52). In an unreported study comparing attitudes of 10th grade females who had spent most of their educational life in single or mixed sex classrooms, females from the mixed sex classrooms exhibited significantly more positive attitudes toward mathematics (Fennema-Meyer, Note 2).

Before single sex classrooms are embraced as a panacea for females' educational equity, careful examination must be done concerning long term effectiveness of such programs. In reality, this may be a partially non researchable problem. No one can foresee the implications for females 50 years from the present time of being isolated in their mathematical training. Because of what has happened to females as well as blacks over the last century, single sex classroom school organization must be approached with caution.

## Conclusions

Although many believe that males achieve at higher mathematical levels than do females, this is not true for all groups of females and males. Few sex-related differences in achievement are evident before high school. When the number of years of mathematics study is equated, few differences in achievement are found during or after high school. It is true that more females than males elect not to enroll in math courses during high school or post-high school years. The critical problem in providing equity of mathematical education for the sexes is to make sure that equal numbers of females and males study mathematics beyond minimum requirements.

Although this is not a simple task, the first step has been taken, that is, national recognition of the problem. Furthermore, there is enough convincing evidence from recent research to suggest that educational policies in the schools should be able to encourage more female participation in mathematics. We also have enough leads from recent research to suggest that further research efforts could be extremely fruitful in helping us to better understand differences in participation and in suggesting alternative policies for making participation in mathematics more equitable.

It is important to realize we are beginning to understand the cognitive, affective and educational variables that are associated with sex-related differences in course election and mathematics learning. For example, with the possible exception of spatial visualization, cognitive variables do not seem helpful in understanding sex-related differences in math. More information about the interrelationships of spatial visualization skills and mathematics learning is essential because of the increasing reliance on spatial attributes in school mathematics programs and the strong evidence that there are sex-related differences in spatial visualization. Particularly critical is the question of how spatial skills are used in mathematical problem solving and if this use differs by sex.

Research also implies that affective variables appear crucial in explaining sex-related differences in mathematics study. Although much of the previous research which has investigated attitudes toward mathematics is flawed, present literature indicates that two affective variables are important: perceived usefulness of mathematics and confidence or anxiety in learning mathematics. In addition, the domain of variables related to stereotyping mathematics as a male domain seems particularly crucial. More information is critically needed on developmental patterns of affective variables, stability of such variables and effective ways of increasing positive attitudes toward mathematics.

Educational variables which appear helpful in understanding and ultimately eliminating sex-related differences in mathematics can be categorized into variables dealing with learners, teachers and school organization. Although little information is available that is precisely concerned with the relationship between these variables and sex-related differences in mathematics, several tentative conclusions may be stated: 1) Subgroups of females may differ in cognitive and affective variables related to mathematics and may require different educational treatments; 2) Since teachers treat females and males differently, it may follow that this differential treatment affects the willingness of females and males to study mathematics; 3) Because of possible long term ramifications, a mathematics program organized to provide single sex instruction must be approached with extreme caution.

In conclusion, there is much we now know from research about reasons for the unequal participation of females and males in mathematics. First, we know there are differences in mathematics study, starting about the tenth grade (e.g., differentiation of choice begins to be observed as more educational and personal choices are available). Second, we know that there is little difference between females and males in most cognitive variables except for visual-spatial skills. Third, we

not only know that attitudes are important to the choices being made by individual females and that these attitudes come from a variety of sources such as the society, family, teachers, schooling and peers but some attitudes have been identified which are related to on-going participation in mathematics and which discriminate between females and males. Fourth, we also know that there exist some programs that have been successful in changing attitudes. This information also suggests what we still need to know to be more effective in changing females' participation in mathematics. For example, we need more information about when and what affects crucial female attitudes towards mathematics; we must identify those sources that may be most susceptible to change; we need to know more about the comparative effectiveness of certain measures to change attitudes of different females towards mathematics; we need to better understand the real impact of the observed cognitive differences between females and males and the relationships between verbal skills, visual skills and mathematical learning; we need to know more about the actual procedures being used in school mathematics instruction that directly result in inequitable participation in mathematics by the sexes; and we need more examples of successes and failures in actually inducing more female participation in mathematics.

## Recommendations

The body and conclusions of this paper suggest certain recommendations for funding focused on improving females' participation in mathematics. These recommendations are directed toward three areas: 1) Directions for research 2) Intervention or demonstration projects and 3) General considerations to optimize the chances that the money spent will accomplish its goals, i.e., to improve females' participation in mathematics as soon as possible and to ensure that females will continue to be involved with mathematics.

## Directions for Research

1. Well designed studies which will provide knowledge about variables related to females' decisions to study or not study mathematics beyond minimum requirements should receive highest priority. Each study funded should deal with a population of specified characteristics so important conclusions can be generalized to groups with similar characteristics. Since a tremendously complex interplay of affective, educational, societal and possibly cognitive variables affect the decision to study mathematics, many different types of studies are necessary.

2. Studies funded should deal directly with mathematics instruction and learning. The importance of investigations made within the realm where behavior is to be changed can not be over emphasized. Results of such investigations are immediately generalizable and will provide needed information for planning effective intervention programs to increase females' participation in mathematics. The projects to be funded should rest on sound theoretical foundations with translation into action or deeper understanding of the educational realm as the goal.

3. Some studies funded should deal directly with the relation between verbal, visual and mathematical skills, especially as they relate to female-male differences. Stability of relations across many developmental levels as well as patterns of development among these cognitive variables should be investigated.

## Intervention Projects

1. Carefully described action programs designed to increase specific target populations of women's participation in mathematics should be funded. Such programs must have accurately described features and well defined target populations so that the programs can be replicated with similar populations. There are currently many intervention programs in operation.

However, there is pratically no generalizable knowledge about effective intervention, except for the fact that it is extremely idfficult. Each program funded should rest on sound theoretical bases, be designed for well identified sub-populations and have well described features. Any intervention must be planned considering many affective and cognitive variables in the complex environment of schools. Learner characteristics, teacher characteristics, mathematics content, instructional method and classroom organization must be considered.

2. Each intervention project which is funded should include plans for extensive evaluation. Included must be long term evaluation as well as short term evaluation. A permanent change in behavior of females toward mathematics must be the goal of any intervention program and this change can not be assessed immediately following participation in such a program. Critical long range evaluation points should be identified. Plans for gathering data at those points must be included and funds must be made available to ensure that the true effect of interventions can be ascertained. In addition plans for evaluation should include provision for assessing unintended as well as intended consequences.

3. Projects funded should originally be small scale programs. These programs should be located in a realistic field setting so that their effectiveness and social impact and transferability to larger projects can be ascertained.

4. Projects funded should include a diversity of target groups in addition to the females directly involved. The opinions of parents, teachers, counselors and in particular male peers make a direct contribution to a female's decision to study mathematics. Unless such significant others' opinions of females as learners of mathematics is positive, it will be extremely difficult, if not impossible, to change females' traditional patterns of behavior toward mathematics.

Basic Considerations

1. Interdisciplinary teams of scholars as well as practitioners should be actively involved in funding decisions as well as funded projects. Two groups of scholars must be specifically involved, i.e., those concerned with the study of women and mathematics educators. Mathematics education is a specialized field which includes research scientists as well as developers and implementers of curriculum. People from this field can most effectively investigate current mathematics learning as well as make effective plans for changing mathematics curricula. However, mathematics education scholars must be joined by scholars who have demonstrated their concern for achieving equity for women. It is this latter group who will be particularly helpful in identifying specific areas of concern.

In addition, scholars from other disciplines such as psychology and sociology would provide breadth of knowledge which would increase the possibility of change. Since the major changes which will achieve equity in mathematics for females will have to take place in schools, practitioners who understand the parameters of effecting educational change must also be involved in decision - making and carrying out of all projects.

2. A substantial portion of funding should be available for longitudinal studies and long term projects. The importance of longitudinal studies for understanding sex-related differences in mathematics and their related variables is evident. Such studies could provide information about stability of important variables within an individual as well as identify which variables are of prime saliency at various developmental points. Long term evaluation of all projects should be included.

3. Chains of related inquiry must be funded. All too often, both in mathematics education and research on women, isolated studies and projects are done. Isolated pieces of information that result from isolated activities will not effectively help women gain equity in mathematics. Only when profitable studies and effective projects are followed by additional studies and projects will we build enough knowledge to effect change.

[illegible] Studies and projects using nontraditional as [illegible] methodologies should be funded. [illegible] funded should utilize case study, clinical [illegible] observation methodologies. Such method[illegible] [illegible] the opportunities for generating new [illegible] and knowledge. The subtle and difficult to [illegible] curriculum, that says mathematics is not [illegible], has not and can not be identified with [illegible] research methodologies. Observational and [illegible] strategies are needed in order to locate and [illegible] communication patterns and messages from peer to [illegible] teacher to learner, and curriculum to learner. [illegible] we know that behavior is extremely difficult to [illegible] innovative ways to change females' behavior in [illegible] must be encouraged by funding.

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III

# EFFECTS OF BIOLOGICAL FACTORS ON SEX-RELATED DIFFERENCES IN MATHEMATICS ACHIEVEMENT

By

Julia Sherman

Women's Research Institute of Wisconsin, Inc.

## Introduction

The task of this paper is to present a critical review of the literature relevant to the effects of biological factors on sex-related differences in mathematics performance, with recommendations for further research. The paper is organized to consider very briefly some historical, definitional and conceptual issues before proceeding to the literature review. The review will be organized around hypotheses which have been advanced to explain differences in male-female mathematics performance. The review is followed by an annotated bibliography of the most widely cited, important and relevant studies.

### Beliefs about Women's Intellectual Performance

It is widely believed that women are not so intellectually competent as men (Broverman, Vogel, Broverman, Clarkson & Rosenkrantz, 1972; Fernberger, 1948). It is especially believed that women are inferior in mathematics and science. These beliefs, which have been common throughout recorded history, have accompanied by disbelief when women perform well intellectually. Such performances are often attributed to luck, or to grinding hard work, but not to ability. These distorted attributions occur in men and women alike (for a review see Frieze, Fisher, Hanusa, McHugh & Valle, in press). The belief that women are not so intellectually competent as men is also accompanied by the belief that women shoud not be as intellectually competent as men (for a sample of historical views, see O'Faolain & Martines, 1973). The essense of this view seems to be that a woman should not threaten the status of a man. Hence intellectual competence or even excellence is permitted so long as as male status is not threatened, e.g., women can do an excellent job of helping men.

With the increasing social importance of science in the 19th century, scientific explanations for supposed female intellectual inferiority became popular. One of the most commonly believed was that women were intellectually inferior to men because their brains are smaller (Ellis, 1908). Subsequently brain size, in the normal ranges, has been found to be uncorrelated with intelligence. Another common belief was that the womb draws energy away from the brain (Clark, 1873). Not only did this view attempt to account for inferior female intelligence,

but it also contained the implications that intellectual work would damage women's reproductive capacity and health. In this way it served as a rationale to keep women away from education and intellectual work "for their own good."

In the United States in the 1970's views of women's inferiority intelligence and in status have been challenged by a call to the ideals of our constitution and democratic heritage of equality of opportunity. It is well, however, to remain cognizant of the perspective of history. Biological explanations of female intellectual inferiority which now seem foolish have been well respected scholarly views during their time (Shields, 1975).

## Is There a Sex-Related Difference in Mathematics Performance?

While the task of this paper is to consider biological explanations for sex-related differences in mathematics performance, it should be realized that the explanations may be explaining a nonexistent fact. As sex role behaviors have been less rigidly defined and enforced, sex-related differences have decreased and in many instances they have not been demonstrated (Maccoby & Jacklin, 1974). While many researchers continue to report sex-related differences in mathematics performance found differences in favor of males in only 1 of 9 schools before the 9th grade and 2 of 4 schools in grades 9-12 (Fennema & Sherman, Note 1, Note 2). When differences were found, they were small and, contrary to common expectation, they did not increase with the difficulty of the material. It could be argued that the reason sex-related differences were not always found at the high school level was that the girls were a more select group than the boys. This argument, however, does not explain why students at two high schools showed sex-related differences in math performance while students in two other high schools did not although the selection procedure was the same in all four schools. Demonstrated school variations in sex-related socio-cultural factors offers a more likely explanation than that two samples of girls were somehow more intellectually select than the boys. It is beyond the scope of this paper to review the literature on sex-related differences in mathematics performance, but it should be noted that hypotheses to explain sex-related differences in mathematics may be attempting to explain a phenomenon which ultimately is found not to exist.

## Aptitude versus Achievement

Questions related to such terms as "aptitude," "ability," and "achievement" have been discussed at length (Green, 1974). "Aptitude" and "ability" have been variously defined. "Aptitude" has even been defined to include motivational and attitudinal factors (Carroll, 1976), but it will not be so used in this review. The terms "aptitude" and "ability" carry connotations of being inherited while the term "achievement" does not but is specifically defined as being learned. Problems have arisen with (a) the use of achievement tests of such generality as to be similar to ability tests; (b) misuse of tests and terms in ways increasing social risk to certain groups. For example, using ability

tests as the only pre-post measures of intervention programs; finding sex-related differences on achievement tests between groups unequal in mathematics background and from this implying the sexes are different in mathematics ability. Because of their implications, the terms "aptitude," and "ability" will be seldon used. Terms of potentially lower social risk will be preferred, e.g., "skill," "performance," "disposition" (Humphreys, 1974). These terms are to be considered neutral in that no assumption is made that they imply inheritance or lack of it. Only cognitive rather than affective factors affecting math performance will be discussed.

Nature of Mathematics

Mathematics is a great variety of intellectual activities of varying complexity, involving diverse cognitive skills. Concern about women's performance in mathematics focuses upon their performance of advanced mathematics. It is generally acknowledged that girls and women can do as well or better than boys and men in computation and that few if any sex-related differences in mathematics performance are found up to about age 13 or until high school (Maccoby & Jacklin, 1974). Considerable focus has been placed on possible sex-related differences in what has been variously called "quantitative reasoning," "arithmetic reasoning" and "mathematical problem solving." The latter term will be used in this paper. Mathematical problem solving can be simply defined as finding the solution to a stated problem involving mathematical ideas, for which one does not possess a standard solution. Emphasis will be placed on performance of mathematical task at least at the high school level and excluding computation. It can be assumed that when "differences" are described they are in favor of males unless otherwise indicated.

Scope of the Paper

Biological factors to be considered. Much scientific writing in the areas of sex-related cognitive differences does not explicitly relate the hypothesized causes to genetic differences. However, one can infer that genetic differences might be involved in such hypothesized influences as hormonal differences, rate of maturation differences. In some instances one assumes the authors intend to imply an ultimate genetic basis for their hypothesis, but this link is not explicit and other interpretations are possible. For example, discussions of the hypothesis that there are more male geniuses and more male mental defectives, usually implied rather than explicitly stated that a genetic reason accounted for the difference. An explicit genetic account of this hypothesis has been supplied more recently (Lehrke, 1972, 1974). Another example are hypotheses related to brain lateralization (Buffery & Gray, 1972); Levy-Agresti & Sperry, 1968). Sex differences in brain lateralization are presumably genetic though not always explicitly so described and they could be attributed to other causes.

On the other hand, hypotheses of direct genetic influence (e.g., Stafford, 1972) do not spell out the mechanisms by which the genes might have their effect. While this does not, by itself, invalidate such an

hypothesis, an hypothesis with an imaginable mechanism is more plausible than one without. For example, it is not easy to imagine a mechanism for sex-related differences in the inheritance of a gene for mathematical problem solving that conforms with the known facts that there are not sex-related differences in general intelligence and that mathematical problem solving involves a variety of cognitive skills. In general, very little has been written relating the various hypotheses and possible mechanisms.

Most of the hypotheses regarding sex-related differences in cognition, do not directly concern mathematics achievement. Some have to do with general ability including presumably math ability; one hormonal hypothesis has to do with performance of simple repetitive tasks as opposed to performance of tasks involving structuring, restructuring, and several relate to the spatial disposition and/or the spatial disposition compared to the verbal disposition. The possible relevance of general ability to math ability is obvious. Tasks involving structuring, restructuring are thought to be related to mathematical problem solving. Werdelin (1961), for example, thought that sex-related differences in structuring-restr turing were most crucial to cognitive sex-related differences in mathematical problem solving. The relationship of the verbal disposition to mathematics is well accepted, however, the relation of spatial disposition to math achievement has not been so obvious to scholars. This is curious because, from another vantage point, mathematics itself could be viewed as the symbolization of spatial relationships.

Relevance of the spatial diposition to mathematics. Since mathematics involves such diverse activities as computation, mathematical problem solving, integration and topology, it is obvious that the quality and type of cognitive skills involved differs with the task, spatial skill being important to integration and topology, for example, but not to computation. It has been hypothesized that spatial visualization is related to mathematics performance and that sex-related differences in space perception might partly account for sex discrepancies in mathematics performance (Sherman, 1967; Smith, 1964). In a large scale investigation of this question, for both sexes, both vocabulary and spatial visualization were found to be moderately correlated (Typically, .40 to .40) with mathematics achievement in grades 6-12 (Fennema, Sherman, Note 1, Note 2). The correlations between spatial visualization and math achievement were consistent from subsample to subsample of the overall sample of nearly 2000 students, lending further confidence to the reliability and validity of the results. (For further evidence of the relationship between spatial visualization, and mathematics, see also references in Sherman, 1967 and data in Bennett, Seashore & Wesman, 1966.) The main source of the contrary opinion that spatial visualization is not related to mathematics achievement appears to be Very (1967) who in turn seems to base his opinion on Murray (1949). Murray's data were based only on males and showed a definite contribution of spatial visualization to geometry grades, but less to Murray's geometry test which was, by his own admission, quite verbal in character. Murray's point had been that one cannot assume geometry to be based primarily on spatial skill. In this, of source, he is correct. Memorizing definitions and theorems is not a spatial task, Overall, however, the evidence is very clear that

spatial visualization is substantially related to many forms of mathematics achievement.

Hyde, Geiringer, Yen (1975) found that sex-related differences in mathematical problem solving can be accounted for by sex-related differences in spatial disposition. Furthermore tests of spatial ability, field independence, and mental arithmetic emerge together as a spatial factor. The Fennema and Sherman studies (Note 1, Note 2) showed evidence that sex-related differences in math achievement could be attributed to sex-related differences in space perception, but socio-cultural factors appeared to be more clearly related to sex-related differences than did space perception. The lower female math performance found in one school in grades 6-8 was not associated with lower performance on the spatial tasks but with more negative socio-cultural influences among the girls than the boys. The girls showed less confidence in their ability to do mathematics and boys more than girls stereotyped mathematics as a male domain. The less math was stereotyped as a male domain, the higher the math achievement. Theses correlations were moderate and statistically significant. The same sort of finding was true at the high school level. In the two high schools where differences in math achievement were found, differences on the spatial task were found in only one school while differences favoring males were found in at least half the eight socio-cultural factors at both schools. Moreover a sex-related difference on the spatial task was found at a third school, but was not accompanied by a difference in mathematics achievement. These findings show sex-related differences in space perception are involved in sex-related differences in math performance, but that so are several socio-cultural factors.

However, as in the case of sex-related difference in mathematics performance, some sample have failed to show sex-related differences in space perception (Fennama & Sherman, Note 1, Note 2; Maccoby & Jacklin, 1974; Sherman, 1974). Such negative findings are important because they suggest that the sex-related differences in space perception are not incorrigible. If there were a strong sex-linked, genetic component to sex-related cognitive differences, one would expect the differences to appear in all samples. Like mathematics, spatial perception is stereotyped as a male domain (Stein & Smithells, 1969). It has been hypothesized that performance on spatial tasks is itself affected by sex-typing in that males receive more practice on tasks which develop spatial skills (Sherman, 1967). At the two high schools where sex-related differences in courses taken which could involve development of space perception (e.g., drafting, design) eliminated the differences between the sexes (Fennema & Sherman, Note 2). Consistent with this view, Johnson (1976) found that while females performed more poorly than males in space perception at the beginning of a drafting course, there was not significant difference between the sexes at the end of six weeks. It is, however, beyond the scope of this paper to review the evidence related to socio-cultural factors relevant to sex-related differences in space perception and/or math performance. These topics have been considered only to provide perspective for the review of evidence relevant to the effects of biological factors on mathematics achievement.

The scope of this paper will include: (a) the hypothesis of greater male variability; (b) metabolic explanations of sex-related cognitive differences (gout); (c) hormonal explanations; (d) X-linked hypotheses; (e) explanations based on hypothesized sex differences in brain laterality. Each section will contain a summarizing statement and discussion of research implications. Final sections of the review will provide an overall summary of findings and research implications.

## Greater Male Variability

The extent to which the sexes differ in the dispersion of intelligence has been of interest since the 19th century. Much of the early focus was on the question of genius and why there seemed to be so few women of genius (Terman, 1925). It gradually became clear that not only did women of genius seem scarce, but that there also seemed to be more mentally retarded and defective males than females. Attempts have been made to ascertain if this is true and to explain these phenomena. It is possible that no single explanation will prove satisfactory. For example, different factors may account for the excess of male mental defectives than account for the excess of male geniuses.

While it is again beyond the scope of this paper to review socio-cultural factors, these are clearly involved in the social recognition of genius status. They begin with the valued expectation of male child, his infusion with expectations, his nurture, his status, and the opportunities that then seem to unfold "naturally" for him as a young adult. They involve even the very judgment of creations and intellectual achievements as to their worth and importance. Even the attribution of discoveries is affected by socio-cultural expectations. Are we really to believe that it was Ikhnaton with his grotesquely deformed body, victim of inbreeding, who thought of the concept of one God, or was it his "wife," the beautiful Queen Nefertiti?

Recent interest in the variability hypothesis has been stimulated by the Stanley, Keating and Fox (1974) study of children precocious in mathematics and science, and by Lehrke's (1972, 1974) explanation of the hypothesized greater variability of male intelligence as attributable to X-linkage of verbal ability.

Stanley et al. (1974) attempted to locate mathematically and scientifically gifted children by staging well advertised contests during which the children took various tests measuring their skill in these areas. They located many more precocious boys than girls and their top winners have been male. These authors have been intrigued by this sex-related difference, but have not come to any definite conclusion as to its cause. Astin (1974) explored these data noting many differences between the male and female children in their attitudes and opportunities. For example, boys were given more books and equipment related to math and science; many of the girls' parents were not very interested in their daughter's precocity; in fact, many of the girls' parents did not even plan to send them to college.

Curiously the test scores of 8th grade girls were lower than those of 7th grade girls. Ordinarily one would expect progressive intellectual development with age and grade. These data provided some definite clues of socio-cultural factors affecting this sex-discrepancy in math-science precocity, but nonetheless the findings have simulated the thought that there are indeed more male geniuses than female geniuses.

A very troublesome research problem has had to do with the selection factor. In the Stanley et al. (1974) procedure as in the Terman (1925) study, there was much room for selection bias, a fact these investigators have been well aware of. That is, more boys than girls may present themselves or be presented by others as precocious. This problem was recognized by researchers very early. However, in order to avoid selection bias, one must examine objectively an entire population. Needless to say few investigators have shown the stamina for such an undertaking. Data which is based on less than an entire population are virtually worthless. Maccoby and Jacklin (1974) discussed this issue, recognized the problem of selection bias, but then examined results of selected comparing the sexes in variability. They cited the Stanley et al. (1974) study as evidence that there is indeed an excess of males at the upper end of the distribution of mathematics performance. In regard to verbal skills, they concluded that the evidence was not clear, while their files contained too few relevant studies of spatial skill to make any evaluation. Because of possible selection bias, such comparisons are unjustified and inappropriate as evidence bearing on the variability hypothesis.

Some of the best data and most thorough consideration of this question are very old (by American scientific standards!). Extensive surveys with very little selection bias were conducted in Scotland in 1932 and 1947 (Scottish Council, 1933, 1949). These surveys used objective tests, largely verbal, to survey 11-year old children. Slightly larger standard deviations were found for males (15.92 vs 15.02 in the 1932 survey and 16.68 vs 15.44 in the 1947 survey). Because of the very large samples, these differences were statistically significant. Upon further analysis, this sex-related difference was found to result largely from excess of males with very low scores. Moreover more males had physical handicaps which could have adversely affected performance (Anastasi, 1974).

It does appear that there may be more male mental defectives and some of the excess of male mental defectives may indeed be attributable to the fact that males do not have the protection of a second X. This explanation has been given for the fact that males are more subject than females to a great many maladies (Childs, 1965). The finding of more frequent male mental deficiency, but not greater male superior performance, is consistent with the fact that positive effects are usually genetically dominant and hence found on a X chromosome would be equally expressed in males and females whereas

negative effects tend to be recessive and hence if found on an X chromosome would be expressed more in males than in females (Wittig, 1976).

## Lehrke Hypothesis

Lehrke (1972, 1974) has hypothesized that major intellectual traits, specificially verbal ability, are carried on the X-chromosone. Lehrke based his hypothesis on the observations that (a) Stafford (1961) found evidence of X-linkage for space perception; (b) males are more variable than females in intelligence, and (c) pedigrees of some forms of mental deficiency, especially involving verbal deficit, seemed to fit an X-linked pattern.

However, Stafford's hypothesis for recessive, X-linked inheritance of spatial visualization has been disconfirmed (see section on X-linked hypotheses). Moreover, Lehrke's hypothesis specifically X-linked verbal inheritance. He apparently was unaware of the fact that in Stafford's (1963) dissertation research on which the 1961 publication was based, he concluded that there was no evidence of X-linkage in verbal ability. This is no way obviates Lehrke's (1974) presentation of pedigree data which seem to indicate an X-linked verbal deficit. That is to say, because it is demonstrated that one can inherit mental deficiency in an X-linked way, it does not necessarily follow that verbal ability in the usual or superior groups is inherited in that way. Indeed other evidence indicates that it is not. As Wittig (1976) points out, there is also no evidence for X-linked inheritance of general ability (Erlenmeyer-Kimling & Jarvik, 1963).

The hypothesis that males are more variable in intelligence has already been discussed. There is certainly no clear, scientific consensus that males are more variable in intelligence. One might perhaps get consensus that there are more male mental defectives (Freire-Maia, Friere-Maie & Morton, 1974), though that has been questioned in the past (Nance & Engels, 1972).

## Conclusion regarding Variability Hypotheses

The question of greater male variability is sufficiently difficult to test adequately that it will probably remain an unsettled question. For example, it could be argued that if population studies were done of spatial performance, the greater variability of males at both ends of the distribution would be clear. There is certainly no adequate scientific support for the general proposition of greater male variability in intelligence though evidence for greater male mental deficiency is reasonably convincing. The lack of solid evidence of greater male variability in intelligence and the evidence of socio-cultural factors involved in the status of genius and the precocious child made it hazardous indeed to attribute the apparent sex discrepancies to a genetic basis.

One must also bear in mind that 120 males are conceived for every 100 females and 107 males are born for every 100 females born. Males then proceed to dies off at a higher rate than females until in the third and fourth decades of life the numbers of males and females are about equal. By advanced age, females considerably outnumber males. In this biological sense every comparison of a male and female group is a comparison of a more select male group to a less select female group.

Implications for further Research

As a biological phenomenon, the apparent excess of male geniuses and math precocious males is probably not a very fruitful line of research for investigators interested in increasing the quantity and quality of female participation in mathematics.

## The Gout Hypothesis

The textbook of Hyde and Rosenberg (1976) presents another explanation for sex-related differences in cognition. The presence of gout in men of genius has been noted historically. Gout is a metabolic disease characterized by elevated levels of uric acid in the blood (BSUA) and its deposit as a salt in various parts of the body, notably in the joints or as kidney stones. These deposits can be a source of extreme pain and interfere with function. About the only consolation gout sufferers have is that they suffer from the "disease of genius."

Since some studies have found lower levels of serum urate in women, Hyde and Rosenberg speculate, "We may be seeing here a biological source of the paucity of professional achievement among women...While feminists may not welcome this view, it does suggest the possibility that women in the future may be able to take a pill that will give them the motivation to complete medical school (by raising BSUA levels)!", p. 137. It should be noted first of all that one of the few studies which compared male and female achievement with some attempt to control relevant variables, did not find a difference in achievement (Simon, Clark & Galway, 1967). It should also be noted that Hyde and Rosenberg imply that the reason few women graduate from medical school is that they lack the motivation. It is extremely doubtful that one could empirically support that point. It is interesting to note that there is a certain ambiguity in the hypothesis. On the one hand, one is led to infer that a high level of blood serum uric acid is related to intelligence, i.e., genius, and on the other hand, the implication seems to be that it is related to motivation. Lastly it should be noted that serum urate level is affected by physical activity which has been an uncontrolled variable in most studies.

There appear to be several propositions relevant to this hyposthesis: (a) blood serum uric acid level is related to intelligence; (b) serum uric acid level is related to achievement; (c) these relations are

causal; (d) blood serum uric acid levels are lower in females; (e) serum uric acid relates to intelligence/achievement in the same way in women as it does with men. Each of these propositions will not be examined. (Blood serum uric acide, BSUA, and serum urate will be used with the same meaning.)

Serum Uric Acid and Intelligence

In fact there is no indication of an important relationship between serum urate level and intelligence (Mueller, Kasl, Brooks, & Cobb, 1970). Serum urate level correlated positively and significantly with the Otis test in a group of 817 army recruits (Stetten & Hearon, 1959), but the correlation of .08 is too small to be of much practical importance. Likewise a correlation of .10 was found between the Otis and serum uric acid among high school boys, but no relationship was found between serum urate levels and scores on the Medical College Aptitude Test for medical students (Dunn, Brooks, Mausner, Rodnan, & Cobb, 1963). It seems that if there are fewer female geniuses it is not likely that it is because their intelligence suffers from lack of uric acid.

Serum Uric Acid and Achievement

There appear to be at least some grounds for supposing there is a relationship between achievement and serum urate. Dunn et al. found serum urate levels of 4.77 for over 500 craftsman compared to levels above 5.30 for 76 Ph. D. scientists and executives. Other findings are that high school boys with high serum urate levels participated in more extracurricular activities, received more science awards and were rated higher by teachers on industry, leadership and responsibility.

Using age-sex corrected serum urate values, Dodgy and Mikkelsen (cited in Mueller et al., 1970), found high levels of uric acid among men in professional and executive jobs and low levels among farmers and those in unskilled jobs. Wives of these men did not show the same differences. Executives enrolled in a development program showed serum uric acid levels higher than men in general or than unselected executives from a small community. This seems to suggest that the more ambitious the executive, the higher the urate level. The emphasis seems to fall most correctly on ambition, not intelligence, genuis or achievement.

Brooks and Mueller (1966) found a correlation of .66 between ratings of achievement oriented behavior and serum uric acid levels in 51 professors, but other investigators have not replicated such dramatic relationships (Mueller, et al., 1970).

There is some evidence then of a relationship between uric acid level and what may perhaps best be described as ambition.

Serum Urate Causes Ambition

Certainly this is absurd if baldly stated since it is well known that the biochemical, hormonal substrata of human behavior act in a more in a more indirect, facilitative fashion. However, maximal brain organization and functioning has proven to be a topic of great interest and theories have been advanced as to the best brains and their evolutionary significance (Levy, 1974; Orowan, 1955). Orowan has suggested that uric acid may be an endogenous cortical stimulant. As we have seen, there does appear to be some indication of a relationship between ambition and serum urate, but what is the direction of that relationship?

It is also known that "stress" increases serum urate levels. Very few studies offer any clues to disentangling the causal chain of relationships. Hyde and Rosenberg (1976) state, "The study by Kasl et al. suggests that it is uric acid that influences achievement, since their uric acid measures were taken before achievements were realized and that college attendance (and presumably achievement-related stress) had no influence on changes in serum urate," p. 136. Actually the evidence presented by Kasl, Brooks and Bocc (1966) was not so clear cut. They presented serum urate levels of 1961 high school males who, in 1965, had either completed college ($n$ = 28), attempted college ($n$= 18) or who had never gone to college ($n$ =16). While there was no difference in the high school grades of the latter two groups, there was a significant difference among the groups and between the latter two groups in their serum urate level in high school. This, if replicated, suggests that (a) serum urate level predicts achievement effort when grades are controlled, and (b) the relationships found serum urate and ambition may not be attributable to the immediate stress of the achievement effort. However these data still do not allow us to infer that high uric acid causes achievement. The authors stated that there was no relationship between the amount of change in uric acid level from 1961 to 1965 and pursuing education past high school, but they did not present the data or the results of statistical tests. Furthermore, we have no way of knowing that the ambitious attitudes that led these boys to attempt college were not also present during high school. That is we have no real basis for concluding that the serum urate level caused ambitious behavior. The authors, in fact, make no such inference.

Hence while there is some reason to believe that high uric acid level is related to ambition, it is not at all clear that high serum urate level causes ambition. When these events are eventually untangled, it may very well be that certain specific kinds of attitudes (Graham & Stevenson, 1963) cause rises in serum urates.

Serum Urate Levels Lower in Women

After age ten, most population studies show that the sexes diverge in amount of serum uric acid (Mikkelsen, Dodge & Valkenburg, 1965; Mueller et al., 1970), Differences between the sexes at all age levels from age ten to sixty were found to be significantly different. The

period of greatest increase in serum urate level was adolescence for males, while females showed much less rise then, but a gradual increase at menopause. These findings plus the fact that blood uric acid was significantly lower in a sample of pregnant women have suggested that sex hormones influence serum urate levels, i.e., specifically that estrogens and possibly progesterone have a urate-depressing ability. This is but one of several similarities to level of cholesterol.

Serum Urate Causes Ambition in Women

As the alert reader may have noted, none of the biosocial studies discussed involved women. As already indicated, there is no evidence the serum urate causes ambition, however, even demonstration of a relationship between achievement and serum urate levels has not been established in women. Dunn et al. (1963) did not find a significant association between number of extracurricular activities and uric acid levels in high school girls (Mueller et al., 1970). Gordon, Lindeman, and Gordon (1967) found a significant correlation between serum urate levels and college achievement during the clinical years of a nursing students, but no relationship during the earlier, academic years. In short, it is not at all clear that ambition and serum urate level are related in women. There is very little cogent evidence on this point. It should also be remembered that there is no reason to believe that ambition or achievement motivation in women cannot or does not equally develop even if their estrogens acted as a urate suppressant. Perhaps the estrogens themselves or some other diochemicals serve equally well to stimulate the female brain <u>without</u> so great a risk of developing gout.

Conclusion regarding Gout Hypothesis

Obviously there is no way this hypothesis could account for sex-related cognitive differences. It is extremely doubtful that it is even cogent.

Hormonal Effects on Sex-Related Differences in Cognition

This section will concentrate on the best known theory of a hormonal basis for cognitive sex-related differences, that of Broverman, Klaiber, Kobayashi and Vogel (1968). The theory will be briefly described and then the validity of its assumptions, conceptualization and empirical base will be examined.

It has been hypothesized that (a) there is a negative relationship between performance of simple, repetitive tasks and those complex tasks requiring inhibition of immediate responses to obvious stumuli (cognitive restructuring); (b) females are better than males on simple, overlearned tasks while males are better than females on more complex, restructuring tasks; (c) this negative relationship between performance on cognitive tasks is an expression of the underlying antagonism of the adrenergic (activating) and cholinergic (inhibiting) systems; (d) estrogens are

stronger activators than androgens. This theory has been criticized on numerous grounds (Singer & Montgomery, 1969; Parlee, 1972) without effective refutation (Broverman, Klaiber, Kobayashi, & Vogel, 1969). After examining each of the propositions of the theory in turn, the data bearing on the theory will be examined. Finally some related topics of interest will be examined.

## Negative Relationships between Simple and Complex Cognitive Tasks

A negative relationship between performance of simple, overlearned tasks and complex tasks requiring restructuring has not been demonstrated. Indeed it would be reasonable to expect that facility in the performance of simple tasks could lead to facility in performing more complex tasks. Generally cognitive skills are regarded and found to be positively related. Broverman and Klaiber (1969) argue that the negative relationship between perceptual-motor and perceptual-restructuring tasks is masked by general intelligence, "g" variance. Some indirect evidence is presented to support their case, but no clear unambiguous demonstration has been forthcoming.

The authors' usage of the opposing terms "activation" and "inhibition" as short-hand for what they say they really mean, e.g., activation of either the adrenergic or cholinergic systems (Broverman, et al., 1969) lends a specious face validity to their opposition of the intellectual tasks each is supposed to benefit. The usage is specious since the adrenergic and cholinergic systems are not truly opposite in action. In short, there is a conceptual slide from common meanings of activation and inhibitition to technical use of the terms as representing activities supposedly of different parts of the central nervous system. The fact that no such parts have been identified in the central nervous system also does not engender confidence in the theory.

A further source of confusion lies in the conceptual slide from a dynamic to a status prediction. That is, while it is intuitively appealing to imagine performance of simple repetitive tasks being facilitated by a high level of activation, and impulsivity impairing the greater reflection that might facilitate performance on a perceptual-restructuring task, this does not necessarily translate to a permanent characteristic of people, nor to a characterization of cognitive differences between the sexes. That is, people arenot blind respondents, but learn and are taught appropriate strategies for dealing with different tasks. A higher level of activation, even if demonstrated, does not necessarily lead to impulsive, unreflective behavior.

## Females Better on Simple, Males on Complex Tasks

The authors have lumped together a variety of tasks on which females perform better, e.g., color naming, work fluency, digit symbol and even verbal function, including reading, and called them simple, overlearned tasks. Conversely the tasks males have been thought

to do better on, notably spatial tasks, were put together and called complex tasks requiring restructuring. This division is grossly arbitrary and inconsistent with any other researcher's view (Parlee, 1972). It illustrates what might be called the Yingle, Yangle fallacy, a tendency to false dichotomies stemming from an adherence to Yin, Yang thinking, belief in the Female and Male principles. (Its parents, the Jingle, Jangle fallacies involve equating or not equating tests because the names sound the same or different.)

The authors' characterization of the chief difference between the tests differentiating the sexes as attributable to the amount of restructuring required has been shown to be incorrect. Maccoby and Jacklin !1974) pointed out that females generally do better on anagrams which is a restructuring task. It should also be noted that: (a) the Broverman et al. dichotomy does not correspond with the division of cognitive tasks into left or right hemisphere tasks, and (b) the Broverman et al. theory cannot account for sex-related cognitive differences before hormonal differences between the sexes are apparent.

Some idea of the cross-species evidence Broverman et al. adduced in support of their theory may be gained from the following quotation: "Human females exceed human males on simple perceptual-motor tasks. The same sex difference in analogous behaviors, that is, spontaneous motor activity is true in rate." (Broverman et al., 1968, p. 37).

## Adrenergic and Cholinergic Antagonism

Broverman et al. (1968) have oversimplified and inaccurately stated physiological facts: equating sympathetic and parasympathetic with adrenergic and cholinergic systems; setting up false and inaccurate antagonisms between the systems; drawing inferences from the peripheral nervous system and applying them to the central nervous system, and selectively presenting evidence of cholinergic action (Singer & Montgomery, 1969).

Broverman et al. (1968) indicated that the best evidence of hormonal effects was the fact that the concentration of choline acetylase, an enzyme involved in regulating the amount of acetylcholine available as a transmitter substance, rises in the hypothalamus of castrated rats. Presumably less testosterone means more inhibiting effexts. Acetylcholine, however, is a transmitter substance in the preganglionic (central) portion of the sympathetic, not the parasympathetic (cholinergic) nervous system as they state. Hence the factual basis of their physiological information and hypothesized mechanism for the role of sex hormones in physiologic antagonism is false (Parlee, 1972).

## Estrogens Stronger Activators

No direct evidence of the greater activating effect in humans of estrogens compared to androgens was presented. The technical difficulties and possible confounding factors for providing adequate data for such an inference are considerable (Bleier, Note 3). For example, the brain

itself can convert androgen to estrogen (Ryan, Naftolin, Reddy, Flores & Petro, 1972; Weisz & Gibbs, 1973). Because of this one cannot be sure that androgen effects are not estrogen effects. For example, an anti-estogen (but not anti-androgen) drug blocked the effects which had been induced with androgen injections for acyclicity of gonadotrophin release (Doughty & McDonald, 1974) and for lordosis (Whalen, Battle & Luttage, 1972). In the face of so much unknown, it is premature to draw conclusions about central nervous system effects of estrogen compared to androgen.

Empirical Findings

Broverman et al. (1968) cited many studies to indicate that drugs which activate/depress the sympathetic nervous system improve/depress performance on simple, well learned tasks. This evidence seemed adequate. However, the evidence cited in relation to complex, restructuring tasks was not at all adequate having to do with rat performance, performance of mental defectives and/or performance on tasks not clearly complex (counting backward, mirror tracing), nor were the reports of human studies numerous (Parlee, 1972). There was also no discussion of the hypothesis of the interfering effects of anxiety, an alternative hypothesis which could explain some of the few remotely relevant human studies they presented (Unger & Denmark, 1975). Some creditable evidence was presented showing a relationship in human males between performance on simple, repetitive tasks and androgen level. This hardly sufficies to support the theory. In fact, it presents the paradox that the most "masculine" men did better on the so-called female task than the more "feminine" men.

Much of the empirical data in support of the Broverman et al. (1968) theory is based only on male subjects, the clearest results being obtained with adolescent male subjects leading to the possibility that results may be confounded with level and/or rate of maturation effects. Most of the correlations between physical attributes and cognitive measures were not statistically significant and, of course, a certain number of correlations can be expected by chance. On the whole, because of these problems, these results can only be considered suggestive for males and do not apply at all to females or to sex differences (Broverman, Broverman, Vogel, Palmer & Klaiber, 1964; Broverman & Klaiber, 1969).

Petersen literature review. Probably the most relevant study is that of Petersen (in press). In addition to presenting her own research, she reviewed evidence relevant to hormones and cognitive functioning. Some of the evidence Petersen cited in support of the general proposition of a relationship between hormones and cognitive function is now irrelevant becuase of the disconfirmation of the X-linked hypothesis of the inheritance of space perception (see section on X-linked hypothesis). Data from persons with chromosomal/hornomal abnormalities were also cited in support of the link between hormones and cognitive pattern, but data from these sources are not very strong, appropriate or convincing. XY persons who look like females becuase their bodies cannot respond to

their normal male levels of androgen were said to show a "female" performance on cognitive tests, i.e., higher performance on a simple repetitive task (Digit Symbol) compared to performance on a perceptual restructuring task (Block Design). While presumably one is to infer that lack of androgen response and upopposed estrogen leads to "female" type performance, there are many unconsidered points: (a) not mentioned is the fact that in the same study, XY "females" reared as "males" did not show the "female" performance (Masica, Money, Ehrhardt & Lewis, 1969); (b) data consist of 15 adults and children combined as one group, a dubious methodology; (c) even the XY "females" performed at normal or above normal levels of spatial tasks; (d) there is not "normal" control group. Some groups of "normal" males also show a "female" pattern. In short, the "female" pattern is a very relative finding.

Another finding Petersen (in press) cited in support of the hormonal-cognitive pattern link is a study of cognitive function in ten males physically feminized at a very early age by a kwashiorkor-induced endocrine dysfunction (Dawson, 1967, 1972). Dawson interpreted the study as supporting the effects of socialization experience on cognitive patterning, but Petersen was impressed with the possibility that the results indicate the effects of estrogen on cognitive function. Compared to a normal control group, the kwashiorkor males showed significantly lower scores on two spatial tests and a significantly higher score on a verbal test. It should be noted that this was a verbal test, not a simple, repetitive task. It is sometimes said of this study that the kwashiorkor males showed a typical "female" pattern in supposedly having lower numerical scores. The difference of .9 points between the controls and the ten experimental subjects is hardly likely to be statistically significant. The Dawson (1972) rate study is very interesting, but suffers a variety of limitations.

It should be noted that kwashiorkor is a very serious protein deficiency disease and may well have caused brain injury and impaired spatial function partly or wholly as a result. In any case, becuase of the small number of subjects, the confounding of hormonal and socialization effects, the use of a verbal rather than a fluency test, and the possibility of brain injury in the experimental subjects, it should be obvious that this study does not provide clear evidence that estrogen facilitates performance on fluency tasks while impairing performance on complex perceptual tasks, i.e. spatial tasks.

Results of studies indicating that performance on both fluent production, simple perceptual tasks and spatial tasks declines with age were cited as indicating a link between cognition and sex hormones since sex hormones also decline with age (Schaie & Strother, 1968; Schwartz & Karp, 1967; Witkin, Goodenough & Karp, 1967) . It has been well known that performance on such tasks declines with age as they also do with cerebral deterioration. In fact lower scores on such tasks compared to performance on a task such as vocabulary are routinely used to infer that the performance decline can be attributed to decline in sex-hormones is unwarranted. It should be noted that if one takes the Broverman et. al (1968) theory

literally, one would predict that a decline in sex hormones should result in improved performance on spatial tasks. Obviously this is not the case.

Petersen data. Petersen (in press) analyzed data from 35 males and 40 females at ages 13, 16 and 18 from the files of the Fels Research Institute. The measures of fluency were the Digit Symbol subtest of the Wechsler at 18. These are different though acceptable measures of spatial skill, the complex, perceptual task, were the Block Design subtest of the Wechsler for ages 13 and 16 and the Space test of the Primary Mental Abilities test for age 18. Comparison of the sexes showed significant differences in favor of males for the two spatial tests and in factor of females for the Digit Symbol subtest, but not for the Word Fluency test.

Addressing the question of the hypothesized negative relationship between fluency and spatial skill, Petersen performed a principal components factor analysis for each sex. She apparently used the scores from the same subjects at age 18 on the PMS Space and Word Fluency tests. Use of so few measures does not make this a very adequate factor analysis of a very adequate test of the hypothesis. However, the results showed a common, general factor for both sexex and a second factor in which, for males, the spatial tests loaded positively and the fluency tests loaded negatively. For females, however, at age 18, contrary to the theory, the Word Fluency test loaded positively with the spatial tests for females. The results can be said to support Broverman & Klaiber (1969) for males, but not for females.

More adequate factor analyses performed with larger numbers of subjects and with more tests do not offer even this limited support to the negative relationship between fluencey and complex task hypothesized by Broverman et al. A principal component factor analysis for 46 females and 35 male undergraduates of nine variables including the Word Fluency test and three measures of complex perceptual task (Identical Blocks, a test of spatial visualization; Rod-and-Frame test, and Embedded Figures test) found three factors, but none for either males or females which loaded positively for fluencey and negatively for the spatial tasks (Hyde, Geringer & Yen, 1975).

Cohen (1975) performed principal component factor analyses on scores from nine tests of 104 women and 90 men of advanced age. The test included the Word Fluency test, the PMS Space test and Gottschaldt Hidden Figures Test. Again three factors were found and again the factor loading the Word Fluency did not have negative loadings for the

complex perceptual tasks for either males or females. In fact, the factor for males with the highest loading (.54) for Word Fluency had a loading of .79 for PMA Space and .74 for Hidden Figures. In view of the fact that neither of these studies support Peterson's findings, it seems unlikely that they are valid.[1]

In order to study the relationship between cognitive performance and hormones, Petersen (in press) made ratings of physical characteristics from nude photographs of the subjects. Three of the physical measures were bipolar ratings from extreme feminine to extreme masculine. They were: (a) amount of fat compared to muscle; (b) overall shape, e.g., shoulder breadth, and (c) size of breasts compared to size of genitals. These presumably reflect the relative influence of estrogens compared to androgens on physical development. The fourth measure was of extent of public hair which presumably measured androgen effect. The Broverman et al. (1968) theory, however, dealt with the presumably greater activating effect of estrogens than androgens. The measures which Petersen used are probably not the most direct tests of the theory. More direct tests would be measures of estrogens and androgens in the blood, though one would have no idea of what hormones, at what level, were effective in the brain. Even using measures of physical appearance it would seem that some greater clarity could be achieved by <u>separate</u> measures of estrogen and and androgen effects (a la Bem). In this instance, as Petersen notes, measures of estrogen effects for females may not have been very adequate especially since breast size and fat can be influenced by non-estrogen factors.

The relationship between the cognitive and physical variables was tested by a canonical correlation calculated for each age and sex. The correlations were significant for both sexes at age 16 and 18, but not at age 13. The nature of the relationship was that by age 18 for males greater masculinity in physical characteristics, especially muscle development, was related to a pattern of better performance on the PMA Word Fluency test and worse performance on the PMA Space test. For females, by age 18, greater masculinity in physical characteristics, especially in overall build and greater androgen effect as indicated by amount of pubic hair, was related <u>only</u> to better performance on the PMA Space test; these characteristics were not related to performance on the PMA Word Fluency test.

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[1] The matter is really more complicated, than indicated here. These studies rotated matrices which tends to eliminate negative values. Disagreement exists as to whether nonrotated matrices can be appropriately interpreted (Smith, 1964). The question of the negative relation of abilities would benefit from further conceptual clarification and a careful empirical review of existing data.

Just how to interpret these results is by no means clear. They are not accounted for by timing of maturation differences since Petersen reanalyzed her data and did not replicate the Waber (1976) findings (see section on differentiation at adolescence). The male results are consistent with some of the Broverman et al. (1968) contentions and findings: positive relationship between physical masculinity and fluency; negative relationship between physical masculinity and spatial skill, though the finding at age 18 of no difference between the sexes in fluency was not consistent with the Broverman et al. theory. The 18 year old, female spatial results are consistent with the Broverman et al. theory in that low masculinity, presumably high estrogenicity, in females related negatively to spatial performance. One wonders, however, what Petersen's androgyny measure means in terms of central nervous system hormone influences. Apparently we are to suppose that estrogens cancel out androgen effects so that there is less activating effect in androgynous males and females than in either high estrogen females or high androgen males. Thus one would conclude that high androgen or high estrogen, but not high estrogen and high androgen (as they cancel out) impairs spatial performance. This is a possible, but not a necessarily true interpretation of the relationship of these hormones to each other.

If spatial performance is impaired by a high estrogen level, one would expect spatial performance to vary with phase of menstrual cycle. Estrogen drops during the premenstrual phase of the cycle, remains low during menstruation and rises again after menstruation (Sherman, 1971). In general, however, many studies have failed to demonstrate significant menstrual cycle effects on intellectual performance. Specifically, Englander-Golden, Willis and Dienstbier (Note 4) found no difference in performance on the Space Relations test of the Differential Aptitude tests between the premenstrual, menstrual and mid-cycle (days 7-13) phases. Also, contrary to what one would expect from the Broverman et al. (1968) theory, there was no menstrual phase difference in performance on the Digit Symbol subtest. Increased levels of estrogen did not improve performance on this perceptual fluency task.

Zimmerman and Parlee (1973) also found no menstrual phase differences in digit symbol performance. The results of these studies are totally contrary to the Broverman et al. (1968) theory (though this was not the focus of either study) and disconfirm it at least for estrogen effects in females and as an explanation of cognitive sex-related differences.

How then might one account for the observed empirical relationships: a negative relationship between physical masculinity and performance on complex-perceptual tasks in males; a positive relationship between physical masculinity and performance on complex perceptual tasks in females? Another possibility Petersen (in press) considered is that humans, like lower species, pass through a critical period in prenatal development when the sex hormones "prime" or organize the brain for later sex-related behavior (Money & Ehrhardt, 1972). This view would require that physical masculinity in males

and females at age 18 would be related to prenatal organization of the brain. (See section on prenatal hormonal cognitive influences.) Petersen suggested that such prenatal organization might affect brain laterality and hence presumably spatial function. More more information will have to be accumulated, however, before even tentative conclusions of this nature are warranted.

Conclusions regarding Broverman et al. Theory

This theory contains factual errors, conceptual distortions and has very little empirical support. In its present form it could not possibly account for cognitive differences between the sexes. Available data confound too many factors to permit interpretation.

Pre-natal Hormonal Effects on Cognition

There is evidence in lower animals that during a species - specific critical period very early in development, sex hormones differentially organize the brain for male or female behaviors (Money & Ehrhardt, 1972). Normally in humans some fetal androgenic substance is responsible for the sexual differentiation to maleness. Without this substance, a female form will develop. In lower animals, during the critical period, exogenous doses of androgens were found to increase certain behaviors such as aggression, but estrogen was also found to produce similar results (Edwards & Herndon, 1970; Payne & Swanson, 1972). The sex hormones are all closely related chemically and can change from one to the other in the body. As already mentioned, androgens can be converted to estrogens in the brain so what seems to be an androgen effect may, at another level, be an estrogen effect. This is the reason for many so called paradoxical effects of sex hormones and for a certain amount of confusion among those reviewing and reporting research findings.

For example, Dalton (1968) found that the daughters of mothers who had received injections of natural progesterone during pregnancy (to guard against miscarriages) scored significantly higher than a group of controls on general aptitude tests. (Progesterone is one of the main hormones of pregnancy.) Ehrhardt and Money (1967) reported that fetally androgenized girls had unusually high IQ's. These girls had been "androgenized" because their mothers had mistakenly continued to take birth control pills during pregnancy. These pills contained synthetic estrogens and progestins. The infants were thus "androgenized" and masculinized by what are quintessential female though artificially manufactured hormones. The babies had external genitalia with male-like aspects. In some instances these data, including even the Dalton results, ended up being reported as male hormones lead to high IQ's. Since Baker and Ehrhardt (1974) found that the higher IQ's of the fetally androgenized girls were also present in the normal sisters of the affected girls, it appears likely there was not true effect. The high IQ's were probably an artifact of the generally more sophisticated level of these families. The results of the Dalton study, however, still remain to be explained. If there is any true, replicable effect, however, it may have to do with the amount of sex hormones rather than

the kind of sex hormones.

One study has reported that prenatal exposure to exogenously administered estrogens and small amounts of progestins, was associated with lower performance on the Embedded Figures test, a spatial test (Yalom, Green & Fisk, 1973). That this effect can be attributed to the prenatal estrogen organizing the brain in some female way is highly unlikely since at least half of the subjects received the hormones late in gestation, a time surely beyond the human critical differentiating period. Furthermore, one would have no idea what the actual type or level of hormone action was in the brain as a result of these hormones. The subjects were the sons of severely ill diabetic women, so ill they required daily medical care. Such children are notoriously subject to brain injury and birth complications. The mothers received the hormones to help prevent miscarriage. Furthermore the nineteen experimental subjects were not significantly different from the fourteen subjects in the normal control group, but a borderline, $p<.10$, effect on a one-tailed test was demonstrated when the experimental group was compared to a group consisting of the normals and eight children of diabetics whose mothers had not had hormone therapy. (These mothers had less severe diabetes.) The result was not replicated when a sample of twenty, six year olds whose diabetic mothers received hormone therapy was compared to a sample of seventeen normals. In brief, because of use of abnormal subjects, too few subjects, inadequate probability levels and failure to replicate, this study does not demonstrate any effect of prenatal female hormones on space perception.

## Cyclic/Psychic Reactions: A Confounding Variable

It has been suggested that the somewhat lower scores on spatial tasks of many female groups when compared with male groups might be accounted for by a sub-group of females showing decrements in performance in reaction to menstrual cycle phase (Englander-Golden et al., Note 4). On the surface, this appears to be unlikely because no statistically significant, reliable decrements in intellectual functioning related to the menstrual cycle have been demonstrated (Parlee, 1973; 1971; Sommer, 1972; Zimmerman & Parlee, 1973).

Englander-Golden et al. (Note 4) predicted that only those females who typically repress would show performance decrements during the premenstrual and menstrual phases compared to the mid-cycle phase (days 7-13). These effects were predicted for the Space Relations test of the Differential Aptitude Tests, a complex perceptual task, but not for the Verbal Reasoning test, a complex verbal task, nor for the Digit Symbol test, a simple perceptual, "fluency" task. Dividing the sample ($n=44$) at the median according to their tendency to repress, Englander-Golden et al. found significant interaction effects for menstrual phase and tendency to repress for both the Space Relations and Verbal Reasoning tests. During menstruation, but not during the other phases, women with tendencies to repress scored lower on both tests. No main effects or interactions were found for Digit Symbol.

These results suggest that menstrual cycle decrements in performing complex tasks among a sub-group of repressing women, could be a confounding factor in studies of sex-related differences. Such studies often compare male and female groups differing in many, many respects, e.g., previous training, experience, handedness, history of brain injury, selectiveness, maturity in regard to task, impact of sex biasing conditions, examiner, test items, group pressure. Englander-Golden et al., however, have demonstrated another specific source of error variance in comparing the sexes which may partially account for disparate findings from study to study.

An additional matter of interest is the possibility of intervention to ameliorate the repressive tendencies of these women. A fuller integration of bodily functioning into awareness might improve the intellectual functioning of this sub-group of women.

## Hormones and Behavior

At this point, it is probably apparent that there is very little that can safely be concluded about "sex" hormones and intellectual functioning. Even if relationships between the two are established, one still does not know if the relationship is causal, nor the direction of causality. For example, if it should be replicated that more androgenized girls (more pubic hair, masculine build, wide shoulders) have higher performance on spatial tasks, how can we know that this is the result, as implied, of their "natural" androgen level? How do we know their androgen level is not the outcome of their willingness to engage in more strenuous activities and/or active assertive attitudes and/or that their higher spatial score come from more practice on spatial tasks? It has been known, clinically demonstrated, and accepted that attitudes can affect hormone levels (for some examples, see Sherman, 1971). Our present state of knowledge does not allow any of these possibilities to be ruled out.

## Research Recommendations

Relations between hormones and cognition will continue to interest investigators, but in the near future it is an area unlikely to provide any useful answers to questions of the sources of sex-related cognitive differences.

## X-linked Inheritance of Abilities

The idea of X-linked inheritance of cognitive skills was first proposed in its modern form by O'Connor (1943). Basically the X-linked hypothesis is that high potential is carried as a recessive characteristic on the X chromosome. The term "X-linked" is preferred to "sex-linked" because of its greater precision (Levitan & Montagu, 1971). If the X-linked hypothesis were true, it would mean that many more males than females would evidence high potential. In fact, the

proportion of females showing high skill should be the square of the proportion of males showing high skill. This is because if the charateristic is recessive and carried on the X chromosome, males would need to inherit only one X with the gene on it to evidence the characteristic while females would need to inherit two X's with the gene on it in order to manifest the characteristic. The probability of inheriting two such X's is the square of the probability of inheriting one.

X-linked inheritance has been proposed for mathematical problem solving (Stafford, 1972), and spatial visualization (Stafford, 1961). Stafford (1972) thinks that each ability is carried as a separate recessive gene on the X chromosome (Stafford, Note 5).

Obviously, if true, this hypothesis would go a long way toward accounting for the scarcity of women in math and science and in high level positions in general. Interestingly enough, however, even if true this hypothesis would still not account fully for the scarcity of women since by the estimate of these researchers, one third of the high ability persons would be female and of course women are not found in anything like those proportions in math, science, or in high level positions in general. The X-linked hypotheses have gained considerable acceptance and have now found their way into an introductory psychology textbook as facts (Hyde & Rosenberg, 1976).

This review will be organized to consider first some indirect evidence inconsistent with the X-linked hypothesis and then the results of studies specifically bearing on the validity of the hypotheses of the X-linked inheritance of mathematical problem solving ability and spatial visualization.

Persons Anomalous in Chromosomal or Hormonal Sex

The hypothesis suffers an embarrassment in dealing with the evidence from Turner's syndrome data. Typically persons with Turner's syndrome have only 45 rather than 46 chromosomes. (A few are mosaic with a typical characteristics.) For sex chromosomes they have only one X chromosome and neither a Y nor a second X as is normal. The external genitalia of these persons are female, but they have no functional gonads of any sort and lack proper internal sexual organs. Of course they also do not have gonadal female hormones nor for that matter proper sex hormones of either sex. Various abnormalities are associated with the syndrome, e.g., short stature, webbed neck (Ferguson - Smith, 1965). These persons are often wrongly considered female bacause of their early external appearance, but they are neither male nor female. They usually live their lives as females in a social sense.

If the X-linked hypothesis were true, one would expect that, as for males, the percentage of Turner's patients expressing the superior gene should be greater than for XX persons, since the recessive gene supposedly would have a full chance to express itself. Instead these patients showed a spatial deficit. Shaffer (1962) reported a degree of what he called space

form blindness in these patients, though below normal spatial performance does not appear in every patient. These findings have generally been extended and confirmed (Buckly, 1971; Money & Granoff, 1965). Garron (1970) resolved this contradiction, by adding the additional hypothesis that the gene had not been able to express itself in the absence of proper sex hormones. Bock (1973; Bock & Kalakowski, 1973) accepted this hypothesis and suggested that a minimum androgen level, specifically testosterone, is required for normal spatial ability.

The data that Bock relies upon for resolution of this question is that of Masica, Money, Ehrhardt and Lewis (1969). These authors studied the intellectual functioning of XY males whose bodies are unresponsive to androgens. As a result they are born looking like females and are reared as females. As adults their sex hormone level for estrogen is between that of normal males and females (Federman, 1968). They are less responsive to androgens than normal females. For example, they have very sparse pubic hair. The Wechsler Verbal IQ, 112, of a sample of fifeen of these persons was higher than the Performance IQ, 102. The Performance Scale measures mostly spatial function. From this it has been inferred that the X-linked gene for superior spatial visualization can only be expressed when accompanied by an adequate level of testosterone. It was then further inferred that spatial visualization is not only X-linked, but testosterone-limited (Bock & Kolakowski, 1973). There are many problems with this line of inference: (a) the Performance IQ was very much in the normal range. Who is to say that the Verbal IQ is not higher than it "should" be rather than that the Performance IQ is lower? (b) What would comparison with an appropriate control group show? (c) The article who were reared as <u>males</u> and not as females was quite different, Verbal IQ 117, Performance IQ 119. If the gene expression is dependent on testosterone, why is Performance IQ even in the normal range let alone above normal, PIQ, 119? Normal brain development probably does depend on at least a minimum level of sex hormones, but there is no evidence they need to be androgens nor specifically testosterone.

In general, while there is no question that mental retardation is often associated with sex chromosome anomalies, considerable question can be raised that there is a "space-form blindness;" spatial deficit, or nonverbal deficit (Garron & Vander Stoep, 1969) are more descriptive terms. Some reference has even been made to "numerical" deficit, but the evidence for this is even less well supported. For purposes of comparison, results of studies of intellectual functioning in several groups of sex chromosome/hormone anomalous persons are charted in Table 1. It would be of considerable interest to have been able to include results of naturally occurring XX persons who look like males (Polani, 1972); however, no relevant studies were found. In at least one case, however, such a person has superior intelligence and works successfully as an engineer (Meissner, Note 6).

Table 1

Wechsler Verbal and Performance IQ's of Persons with Sex Chromosome Hormone Anomalies

| Study | N | Genotype | Sex of Rearing | Effective Adult Hormones | VIQ | PIQ |
|---|---|---|---|---|---|---|
| Money & Granoff (1965) | 44 | XO | F | Low estrogen, androgen | 96 | 86 |
| Masica, et al. (1969) | 15 | XY | F | Low estrogen | 112 | 102 |
| Masica, et al. (1969) | 3 | XY | M | Low estrogen | 117 | 119 |
| Money, (1964) | 23 | XXY | M | Fairly low estrogen, androgen | 105 | 88 |
| McKerracher (1971) | 12 | XXY | M | Fairly low estrogen, androgen | 66 | 76 |
| McKerracher (1971) | 20 | XYY | M | Androgen, normal? | 79 | 88 |

As can be seen, all three groups with more or less than 46 chromosomes have an associated mental deficiency. The group with the lowest Performance IQ, which is a fairly good measure of spatial function, is not the XO Turner's syndrome persons, but one of XXY groups. The variation in Performance IQ cannot be accounted for by the assumptions made by Bock (1973) or by Petersen (in press) about androgen effects. Summarizing the comparison of the Performance IQ's one can say (though without benefit of statistical test) that more or less than 46 chromosomes of either kind has an adverse effect on Performance IQ (spatial function) and among those without 46 chromosomes, more androgenes did not improve Performance IQ.

These comparisons demonstrate the difficulties of dealing with discrepancy measures. Looking only at Performance IQ scores, one would not conclude that XO persons have "space-form" blindness. Use of discrepancy analysis has several problems. Various factors affect verbal and spatial function. High verbal performance tends to be sex-typed female, but at a broader level, acquisition of a certain level of verbal skill is culturally more emphasized, trained and important to overall achievement and status to society than is acquisition of spatial skill. As a result, even in instances when the entire left hemisphere is removed in infancy, adequate verbal skill develops in the right hemisphere through probably at the expense of spatial skill (Kinsbourne & Smith, 1974). Perhaps because of the cultural ascendency of verbal skills, brain injury is more sensitively detected by deficits in spatial function.

It is also well known that delinquent, inpulsive persons have poorly developed verbal skill in comparison to their performance skill (McKerracher, 1971; Wechsler, 1958). The interpretation given to this observation is that they have not been willing to benefit from their cultural opportunity to learn. Some of the variation in verbal skill then may be based on noncognitive aspects of willingness to cooperate, attend, persist, self-correct and hence to learn. These characteristics, which are very much present in XO persons are frequently absent in XXY persons though the suspected propensity of the latter for outright violence proved to be exaggerated.

The data from persons anomalous in chromosomal or hormonal sex lack systematic investigation. One can say, however, that although XO persons have lower spatial than verbal performance, attribution of this pattern of performance to lack of testosterone is unwarranted. Data from persons anomalous for hormonal or chromosonal sex are interesting, however, in their own right and because of their bearing on hypotheses of hormonal associations with cognitive function, a topic discussed in more detail earlier in this paper.

## Tests of the X-linked Hypotheses

There have been various ways of testing the X-linked hypotheses: (a) comparing monozygotic male and female twins; (b) observing the variations in intra-familial correlations; (c) examining score distributions for their conformity to theoretically predicted distributions. The rationale of these approaches will be briefly explained, and then pertinent evidence for the X-linked inheritance of mathematical problem solving and spatial visualization will be examined.

The Lyon (1961) hypothesis, which is widely accepted by geneticists, holds that a few weeks after conception, one of the female X chromosomes is randomly inactivated in each female somatic cell. (This inactivated X chromosome is believed to form the Barr body which serves as a basis for a test of chromosomal sexuality.) Assuming the Lyon hypothesis is correct, and assuming X-linked cognitive characteristics, one would expect greater variability among female identical twins than among male identical twins. Such findings have been reported, but their statistical reliability has not been established (Maccoby & Jacklin, 1974). Consequently this method of testing the X-linked hypothesis will not be discussed further.

Given certain assumptions, such as no assortative mating, it can be predicted that X-linked genes will contribute to the observed variance in continuous characteristics in the following manner: (a) father-son correlations should be approximately zero because males do not receive an X chromosome from their fathers; (b) father-daughter correlations and mother-daughter correlations will be greater than zero because females receive one X chromosome from each parent; (c) father-daughter correlations will be higher than mother-daughter correlations because females always receive their father's only X-chromosome, but may receive either of their mother's X-chromosomes; (d) mother-son correlations should be

equal to father-daughter correlations because in both cases half of the X chromosomal inheritance of the female member of the pair is identical to the X chromosomal inheritance of the male member of the pair. Thus the predictions are: (mother-son $\approx$ father-daughter) $>$ (mother-daughter) (father-son=0).

Another test of the hypotheses lies in the fact that they predict a bimodal distribution of scores. The score distribution should show specific, predictable sex-related differences. The proportion of females showing evidence of high skill should be the square of the proportion of males showing high skill. If the characteristic is recessive and carried on the X chromosome, males would need to inherit only one X with the gene on it to evidence the characteristic while females would need to inherit two Xs with the gene on it to evidence the square of the probability of inheriting one.

X-linked inheritance of mathematical problem solving. Stafford (1963) studies the intra familial correlations on the Mental Arithmetic test of fathers and mothers and their teenage sons or daughters. These data have been presented at various professional meetings and also in a later paper (Stafford, 1972). The correlation pattern did not follow that predicted though it was described as best fitting the X-linked hypothesis; no statistical tests of differences between correlations were run. The father-daughter correlation of .21 should have been about the same as the mother-son correlation of .62. Both should have been higher than the mother-daughter correlation .25. However, the father-son correlation was lower, .08, and close to zero. (Consistent with the assumption of no assortative mating, the father-mother correlation was .07.) Averaging these correlations with some reported by other investigators in the 1920's and 1930's, the correlations fell somewhat more in line with what was predicted, but still not to a convincing degree and still not buttressed with statistical tests (Stafford, 1972). Williams' (19[illegible]) finding of a significant correlation of .31 between 55 fathers and sons of the Wechsler Arithmetic subtest is not consistent with the X-linked hypothesis since, according to the latter, the correlations should be zero. (Correlation between parents was nonsignificant indicating no violation of the assortative mating assumption.)

Stafford (1972) reported score distributions of the Mental Arithmetic Problem test. On the distribution of averaged monozygotic twin scores, he estimated the location of the antimode. Then males and female subjects (300 pairs of twins of various sorts), ages 12 to 18, were sorted as to whether their scores fell above or below the antimode. On the basis of this analysis, Stafford estimated the gene frequency as .43. The proportion of the females above the antimode was .33. However, the hypothesis predicts that this value should equal $.43^2$, or .18 which it does not.

The results of Hartlage (1970) conformed rather well with the X-linked predictions. Bock and Kolakowski (1973) took these results, those of Stafford (1961) and averaged them with some results of their own in order to demonstrate statistically significant differences between the correlations, providing support for the X-linked hypothesis. Bock and Kolakowski (1973) also reported that for the spatial visualization scaled scores over 700 11th grade students, the distributions were better fit by two normal curves than by a single normal curve. That is, bimodal score distributions were found for each sex as predicted by the X-linked hypothesis.

Yen (1975a) studied sibling correlations and within-sex score distributions for four paper-and-pencil spatial tests in a population of 2508 white high school students. Part of the results supported the X-linked hypothesis, but most did not. It is also of interest to note that Yen (175b) did not find any difference in handedness between those of presumably high or low genetype for space perception. This would imply no relationship between the hypothesis of X-linked inheritance and Levy's laterality hypothesis, both presented as explanations of sex-related differences in space perception. However, neither hypothesis appears to have empirical support (see section on Levy).

The report of Stafford (1966) that the scores of the Space Relations test of the Differential Aptitude tests were bimodal as predicted, was not accompanied by sufficient data and/or statistical tests to permit evaluation. Guttman (1974) is sometimes cited in support of the X-linked hypothesis of the inheritance of spatial skill, but provides neither strong nor appropriate support for the hypothesis.

In fact, the empirical support for the X-linked hypotheses has crumbled. Bouchard (Note 8) found that the familial correlations among members of 200 families did not order themselves as predicted by the theory that human spatial visualization is under the control of an X-linked recessive gene. A large scale study in Hawaii of Americans of European ancestry (739 families) and Americans of Japanese ancestry (244 families) found that neither the scores of five different spatial tests nor the score of the spatial factor produced correlations consistent with the X-linked hypothesis (DeFries, Ashton, Johnson, Kuse, McClearn, Mi, Rashad, Vandeberg & Wilson, 1976). Williams (1975) did not find the predicted correlations for the Black Design test in 55 Canadian families. Bock (1967) found the distribution of spatial ability in a sample of 45 families inconsistent with the X-linked hypothesis. Sherman and Fennema (Note 7) did not find the predicted bimodality of scales score distributions of the Space Relations test of the Differential Aptitude Test in a population of over 300 9th grade students. It should also be noted that failure to find sex-related differences in spatial visualization between groups of males and females is not consistent with X-linked hypothesis (Fennema & Sherman, Notes 1 & 2). This accumulation of negative findings disconfirms the X-linked hypothesis of the inheritance of spatial skill. Some of the recent studies are based on much larger numbers suggesting that previous findings were chance variations in small samples.

### Conclusion Regarding X-linked Hypotheses

These hypotheses lack empirical support.

### Implications for Further Research

This does not appear to be a fruitful area for further research.

## Sex-related Differences in Brain Lateralization

What authors mean by "brain lateralization" is often not precisely explained. For normal, righ-handed persons, the two hemispheres of the brain are thought to specialize in somewhat different functions, the left hemisphere specializing in verbal, analytical tasks and the right hemisphere in spatial gestalt tasks (Dimond & Beaumont, 1974; Kinsbourne & Smith, 1974; Milner, 1971). It is also thought that each hemisphere may develop programming favorable to its specific task (Kinsbourne, 1974a, 1974b). Usually when authors speal of differences in "brain lateralization" they refer to the speed or completeness of the establishment of left hemisphere dominance for verbal, analytic function, especially speech, though sometimes they mean dominance of the right hemisphere for spatial, gestalt function. Unless otherwise specified, the discussion will assume right-handedness.

### Mathematics and Brain Lateralization

Most accounts of lateralization of cerebral function are rather simplified. In the main, they locate mathematics in the left hemisphere with verbal, analytic thinking. The only mathematical task noted as located in the right hemisphere is very simple computation (Sperry, 1975). Logically it would appear that females should do as well as males in mathematics since females are generally conceded to do as well or better than males on verbal tasks. As previously noted, however, mathematical tasks are complex; and spatial skill, which is supposed to be a right hemisphere function, can be very much involed in many types of mathematics achievement. Moreover some mathematical tasks may require use of both hemispheres. Bogen and Bogen (1969) suggest that "integrated use of verbal and visuo-spatial thought may depend on interhemispheric communication, including an important contribution from the corpus callosum," (p. 199). They cited many examples from great thinkers, including Poincare and Einstein, about the role of images, difficult to verbalize thoughts and unconsious thinking in creative mathematical work. They emphasize that lack of creativity may stem from "the inhibitory effect, on the appositional source (right-hemisphere), of an excess of propositional (left-hemisphere) thinking" (p.201).

Studies of mathematics and brain lateralization. There have been several studies of mathematics and brain lateralization, but unfortunately for various reasons the results are not easily interpreted. For example,

Bakan (1969) reported that a predominance of right-eye movements (presumably left hemisphere activity) is associated with majoring in a natural science and relatively better mathematical performance on the Scholastic Aptitude Test. The sex of the 22 subjects was not reported. Bakan (1971) made several statements about sex-related differences, but presented no data to support his views.

Harsham and Remington (Note 9) cited studies to the effect that fewer right and more left lateral eye movements occur among women (Day, 1967; Weitan & Etaugh, 1974) though Duke (1968) and Etaugh (1972) found no sex-related difference. Moreover the Weitan and Etaugh (1974) finding is a trend, $p < .10$, over responses to verbal, numerical, and spatial and musical questions ($n$=24 females and 24 males). The theoretical meaning of such a difference, if replicated, is not clear since response to task believed to involve both hemispheres have been lumped together. A greater proportion of lateral eye-movements were made to the right on verbal and numerical questions than to spatial and musical questions, $p < .01$. The numerical questions were computational, which would presumably involve the left hemisphere. Weitan and Etaugh (1973) reported that right-movers scored relatively higher on the Mathematics compared to the Verbal subtest of the Scholastic Aptitude Test ($n$=15, sex unreported). Kinsbourne (1974c) indicated that variations in experimental procedures can lead to varying results with lateral eye movement technique so that some of the discrepant results are doubtless artifactual. At this point it is difficult to conclude anything about sex, laterality, and mathematics or spatial function from these studies.

Hypotheses of Sex-Related Difference in Lateralization

There are several varying hypotheses of differences between the sexes in brain lateralization. These hypotheses will first be briefly described and then the relevant evidence for each will be reviewed. A large number of studies have few female subjects, do not report sex of subjects, or use male and female subjects but not analyze the results separately. For the most part these studies have not been included in this review.[2]

Buffery and Gray (1972) hypothesized that dominance of the left hemisphere for verbal function is attained earlier in girls leading to less bilateral representation of spatial function. Buffrey and Gray believe that bilateral spatial representation is beneficial to superior spatial functioning and hence lack of spatial bilaterality would account for poorer female spatial performance.

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After completing the review, two further unpublished reviews were received (Harris, Note 15; Harshman, Remington and Krashen, Note 16). They do not, however, require substantial alteration of the conclusions reached.

Levy (Levy, 1972; Levy-Agresti & Sperry, 1968) hypothesized that females, like left-handed males, are more likely to have verbal function located in both hemispheres of the brain. It is hypothesized that maximal verbal and spatial function is attained when one hemisphere, usually the left, is specialized for verbal function. Since bilateral representation of verbal function would impair spatial development, females, like male left handers, would have poorer spatial function. Initially, it was thought that verbal function was not impaired because of the cultural importance it receives. Later, Levy (1974) suggested that bilateral verbal function confers an advantage to verbal performance. Maccoby and Jacklin (1974) found the Buffery and Gray and Levy hypotheses in fundamental contradiction. One hypothesis proposes that less lateralization results in development of better spatial skill and the other that more lateralization results in development of better spatial skill. It should be noted, however, that these hypotheses are not direct opposites. Buffery and Gray hypothesized that because males have more bilateral <u>spatial function</u> they are better than females in performing spatial tasks. Levy hypothesized that because females are more bilateral in <u>verbal function</u>, the spatial function they are better than females in performing spatial tasks. Levy hypothesized that because females are more bilateral in verbal function, the spatial function of the right hemisphere is interfered with causing poorer spatial function (Levy, Note 10).

Harshman and Remington (Note 9) suggested that because females mature earlier than males, at young ages females appear to be more lateralized (meaning clear left hemisphere dominance for verbal function)than males. However, when males are fully mature they are more lateralized than females for both verbal, analytic and spatial, gestalt functions.

Sherman (1967, 1971, 1974, Note 11) has presented the "bent twig" hypothesis. She suggested that for more females than males, an early preference for the verbal mode is established. This preference is further developed by a variety of factors including education and cultural influence. The verbal preference is a left hemisphere preference which also is manifested in the greater role of the left hemisphere in spatial function for females. In essense, through a combination of factors, more females than males are likely to take a verbal, analytic (left-brained) approach to tasks. The hypothesis does not assume as incorrectly stated by Maccoby and Jacklin (1974), that because the left hemisphere develops well, the right hemisphere does not or cannot develop well.

<u>Buffrey and Gray Hypothesis</u>

The Buffery and Gray (1972) hypothesis states first that left cerebral dominance for verbal function is established earlier in girls than in boys. Secondly it assumes that because of this earlier left cerebral dominance, females have less bilateral spatial function and thirdly that this causes their spatial skill to be less poorly developed than in the case for males. Each part of this hypothesis will not be examined.

Earlier left hemisphere dominance in females. That left cerebral dominance for verbal function is established earlier in girls is based first of all on data showing more precocious verbal development in females than in males. Relevant studies were most recently extensively reviewed by Maccoby and Jacklin (1974). They cited some studies that did not show verbal acceleration of girls compared to boys. Johnson (1973) found cultural variation in female precocity in reading skill. Females were not more precocious in reading than males in cultures where students were taught by a higher percentage of male teachers. Evidence such as these do not necessarily mean that females are not "naturally" more verbally precocious, but at the very least they indicate that "nature" is not very rigid. They also suggest caution in inferring from apparently invariant psychological "facts" to biological etiology. On the whole, however, the verbal acceleration of girls is generally accepted.

An additional source of evidence for the conclusion that left cerebral dominance is established earlier in females than in males, stems from tachistoscopic studies and studies of dichotic listening. Such studies take advantage of the fact that material presented in a hemi-field of space initially goes to the contralateral hemisphere. For example, the majority of right-handed adults identify tashistoscopically exposed, easily verbalized material more accurately when it is presented in the right visual hemi-field, and hence initially to the left cerebral hemisphere, than when it is presented in the left visual hemi-field, and hence initially to the right cerebral hemisphere. The reverse is found far difficult to verbalize (nonverbal) material. Buffery and Gray (1972) have concluded that there is a linguistic device in the left hemisphere which is more developed in the female than male human brain of the same age so that for females there is a linguistic device in the left hemisphere which is more developed in the female than male human brain of the same age so that for females there is earlier and more complete lateralization.

Kinsbourne (1974a, 1974b) suggested that before age five, the dichotic superiority of the left hemisphere for speech sounds reflects a tendency to orient to the right with speech input, and need not, imply a greater left - than right- hemisphere facility in processing verbal material. If the attentional ascendency of the left hemishere is accomplished with reciprocal inhibition of the right, over time this inhibition will have a cummulative effect on the right hemisphere, i.e., reducing its competency for that particular material. If Kinsbourne is correct, the emphasis should be on the greater attentional ascendency not on the greater competency of left hemisphere verbal function of young females. This conceptualization has a variety of implications, but suffice it to say here that if Kinsbourne is correct Buffery and Gray would need to modify their hypothesis somewhat. Hence, with some reservations, it can perhaps be fairly concluded that left cerebral dominance for verbal function is

established earlier in females.

Earlier left cerebral dominance as a cause of less spatial bilaterality.

Why should earlier left cerebral dominance cause less bilateral spatial representation in females and/or is there any evidence of less bilaterality of female spatial function? Buffery and Gray (1972) did not accept as relevant some findings that the left hemisphere is more important in female than in male spatial functioning (Lansdell, 1962, 1968). On the whole, there is not a considerable amount of data which are inconsistent with their view. Their cerebral dominance for visuo-spatial function occurs later in males than in females. However, Witelson (1976) and Rudel, Denckla and Spalten (1974) report the opposite. Even if it were true that right cerebral dominance for visual-spatial function occurs later in boys, it would not necessarily follow that this results in more bilateral representation of spatial function for males. Harshman and Remington (Note 9) suggest that for adults sex-related differences in lateralization could be quite different than for children. Buffery and Gray presented no solid evidence that males have more bilateral spatial representation than females.

Spatial bilaterality associated with superior performance. Buffery and Gray (1972) presented no independent evidence to indicate that bilateral representation of spatial function is associated with superior spatial function. Indeed, there is some evidence to the contrary (Waber, 1976). Also, in the preschool ages reported as those when males have not yet established right cerebral dominance for spatial function, Coates (1974) demonstrated female superiority in spatial performance. While one can suppose that bilaterality is advantageous, there is no independent direct or indirect evidence that this is so and and some to suggest it is not.

Conclusion regarding Buffery and Gray hypothesis. Buffery and Gray's hypothesis is implausible and lacking in empirical support.

Levy Hypothesis

This hypothesis has not been thoroughly spelled out in print, but was discussed by Maccoby and Jacklin (1974), has circulated in unpublished manuscripts and in informal conversation. Levey (1972) contains at least a partial statement of the hypothesis. Basically the hypothesis is that because women have more bilateral representation of verbal function and because hemispheric specialization results in maximal cerebral organization, females perform more poorly on spatial tasks. Specifically, it was suggested that right-handed females would perform like left-handed males who presumably have bilateral verbal representation. Let us examine each of these

propositions in turn.

Evidence regarding verbal bilaterality in females. The idea that women have more bilateral representation of verbal function is first of all inconsistent with findings that slightly more females are right-handed than males, though this sex-related difference is not clearly established. (Sherman, Note 11). The opinion that females are more verbally bilateral appears to rest partly on observations that females recover from aphasia better than males. Presumably right-handed females becoming aphasic due to injury to the left cerebral hemisphere recover better than comparable males because they have more verbal capacity in the uninjured right hemisphere. This line of evidence is very indirect even if one assumes that adequate numbers of matched males and females have been objectively compared to justify this opinion. For example, since verbal function is often better developed in females than in males and since greater adequacy of performance before damage is associated with better recovery from aphasia, the better development of verbal function in females before injury may account for more complete female recovery. This explanation seems more plausible than assuming there is more verbal capacity in the female than in the male right hemisphere. In any case this inference is not a necessary one.

Other evidence cited in support of verbal bilaterality in females include: (a) studies of temporal lobe surgery on epilepsy patients; (b) studies of women who received unilateral electroshock; (c) tachistoscopic studies; (d) studies of naturally occuring brain lesions and (e) studies of commissurotomy patients.

There are data available from studies of surgery with temporal lobe epilepsy patients. For these patients, part of the temporal lobe is excised in order to reduce the severity of their epilepsy. These studies are particularly valuable compared to studies of naturally occurring lesions since pre-post surgery performance can often be compared; size and nature of brain injuries can be more exactly determined, and the time interval between the brain injury and intellectual evaluation can be more exactly controlled. It was from these studies that the first hint of sex-related differences in brain lateralization emerged.

Lansdell (1961) found that left temporal lobe surgery disrupted the performance of males but not females, on Gorham's Proverbs Test. From this one might infer that females have more verbal capacity in the right hemisphere. However, two later studies by Lansdell do not provide even this slight support for the position of female verbal bilaterality. Lansdell (1962) found a decrement in verbal scores on the Wechsler Intelligence Scale following operation on the left hemisphere but did not report any difference between the sexes in the decrement. Lansdell (1968), also did not find a sex by side interaction for the veral comprehension factor. That is, verbal

comprehension decrement following temporal lobe surgery was the same for males and females.

There have been some studies of depressed women receiving electroshock therapy for their depression in which the shock was applied to only one side of the brain at a time. Using this procedure performance before and after electroshock to either the left or right side of the brain could be evaluated in terms of its effects on intellectual performance. Cohen, Noblin, Silverman and Penick (1968) examined the effect of unilateral electroshock on the performance of 24 right-handed, depressed women. Shock to the left hemisphere resulted in a decrement on a verbal paired-associates task of 16.7; shock to the right hemisphere resulted in a decrement on a visuographic learning task of 4.8 compared to a decrement of 9.5 with shock to the right hemisphere. These data suggest some bilaterality of spatial function, but very little bilaterality of verbal function in women. There was no male group so cross-sex comparisons can not be made. Inspection of the data on both left and right-handed males and females reported by Warrangton and Pratt (1973) does not show evidence of greater verbal bilaterality in females.

Hanny and Malone (1976b) compared tachistoscopic results of females with those of males from a previous study (1976a). The females showed right visual field superiority for verbal material for the 5 second memory interval, $p < .05$, while males showed right visual field superiority for both the 5 second, $p < .01$, and the 10 second interval, $p < .05$. From the facts that males showed left cerebral dominance for verbal function for two experimental conditions and with a more stringent level of probability, it was concluded that females are less lateralized for verbal function than males. No direct statistical comparison of the two sexes was made. The authors' conclusion that their data show less complete lateralization of linguistic function in females is not well supported by their data.

Harshman and Remington (Note 9) cited one tachistoscopic study (Ehrlichman, 1971) in favor of the proposition of greater verbal bilaterality in females and one not supporting it (Bryden, 1965). Lake and Bryden (1976) found no sex difference in lateralization on a verbal dichotic listening task for persons, regardless of handedness, without a family history of sinistrality. For those with a positive family history of left-handedness, males were more asymmetrical than females. These authors reviewed data relevant to the question of greater verbal asymmetry in males. They cited the studies of Carr (1969) and Bryden (1965) as finding no general sex difference, but interaction trends of sex with handedness. Some unpublished data were cited indicating greater lateralization among males, but insufficient detail was given to evaluate whether the differences were significant. Lake and Bryden (1976) unsuccessfully tried to support the case for greater verbal bilaterality in females with data from studies of aphasia. None of 19 right-handed women reported by

Weinsenburg and McBride (1935) had aphasia as a result of right hemisphere damage. Data regarding sex-related differences in brain anatomy was cited. The fact that there were significantly more adult female than male brains with larger cortical speech zones in the right than in the left hemisphere was cited as consistent with the proposition of more verbal bilaterality in females. The difference was not observable in infants (Wade, Clarke & Hamm, 1975). Overall the total number of persons with reversed asymmetry in the sample of 100 was 10, hence the percentage of women involved is small and there is no way of knowing how these results related to other factors such as handedness. Altogether their data are not very convincing of greater verbal bilaterality in females, especially not in right-handed females.

Since females are more viable and verbally facile than males, it is possible that more female infants with injury to the left hemisphere survive and develop verbal function in the right hemisphere. Because of their greater viability and verbal facility and the lesser expectations placed on the female sex, these persons might more likely than comparably damaged males merge with the general population and never be detected as neurologically traumatized persons. Such an hypothesis could account for some of the interaction effects of sex with handedness. Left-handed females in the general population may have a higher proportion of neurologically traumatized persons among them.

Findings that males with left-hemisphere damage score lower on verbal tests than females with left-hemisphere damage are cited as evidence of greater female verbal bilaterality, but these data are not convincing since males may have had lower scores to begin with. More dire direct evidence for bilateral verbal representation in females would be findings that right-hemisphere damage has a more adverse effect on the verbal performance of females than males, but such was not the case in one study (McGlone & Kertesz), though findings were in that direction. Similarly, McGlone (Note 12) reported that the Wechsler Verbal IQ of right-hemisphere damaged males was 106 compared to a Verbal IQ of 99 for females ($n=17$ in each group). Even if one assumes the groups are comparable, the results again are in the predicted direction, but are probably not significant (test apparently not run). Males with left-hemisphere damage scored lower on Verbal IQ than females with left-hemisphere damage. Left-hemisphere damaged males scored lower on Verbal IQ than Performance IQ $p < .01$, while this was not true for females. These facts, however, are less to the point. Citation of these two studies does not exhaust the possible relevant data from brain injured persons, but results of earlier studies were often not analyzed separately by sex nor designed to test for sex-related differences (Sherman, 1974). These two studies, however, present some recent data and serve to illustrate the types of evidence used to support this point.

Using the male proportion above the antimode as the estimate of gene frequency, .43, Stafford tested the observed score distributions of 181 dizygous twin pairs (fraternal twins of the same sex) on the Mental Arithmetic Problems test against the predicted distribution. The fit for males was adequate, however, the fit for dizygous (DZ) female twins deviated significantly ($p < .001$) and the fit for heterozygous (HZ) twins (fraternal twins of different sexes) also deviated significantly ($p < .05$). The fit of these results was described as "not too bad for HZ twins, but somewhat off for DZ girls," (Stafford, 1972, p. 197). These data do not provide support for the X-linked hypothesis, yet Stafford at the end of the article says, "In conclusion, it appears that in general there is an underlying hereditary component for a proficiency in quantitative reasoning which fits the sex-linked recessive model fairly well." (Stafford, 1972, p. 198).

Sherman and Fennema (Note 7) examined the Mental Arithmetic Problems score distribution of 161 male and 152 female, 9th grade students for their fit to X-linked predictions. The proportion of males and females in the upper range of the mathematical problem score resolutions was grossly out of line. The female proportion (.49) was not close to the square of the male proportion ($.53^2 = .28$). While a bimodal score distribution was not rejected for either sex, the means of the two curves were similar and single normal curves were similar and single normal curves were not rejected as fits of the data.

There is no acceptable, scientific evidence of X-linked inheritance of mathematical problem solving.

X-linked inheritance of spatial visualization. Stafford (1969) published part of the results of his dissertation (Stafford, 1963) in which he studies intrafamilial correlations on a variety of cognitive tests. The one which best fit the X-linked hypothesis was Identical Blocks, a measure of spatial visualization. For this test, the ordering of the size of the correlations conformed to theoretical predictions, that is mother-son $\approx$ father-daughter $>$ mother-daughter $>$ father-son=0. Stafford, however, did not statistically establish that the correlations were significantly different from each other. Moreover, he found the magnitude of the correlations consistent with an estimated frequency of .20 while Bock and Kolakowski (1973) found the correlations consistent with a gene frequency of .50. This discrepancy has not lent credence to the findings.

After Stafford's report, Corah's (1965) finding of a higher mother-son than father-son correlation for the Embedded-figures test was sometimes cited in support of the X-linked hypothesis. However, the difference between these two correlations was also not statistically tested and the rest of the correlations did not conform to the X-linked predictions. Corah, himself, did not conduct the study with the X-linked hypothesis in mind.

Other kinds of data that have been incorrectly cited to support the view of female verbal bilaterality include studies of commissurotomy patients (Harshman & Remington, Note 9). These patients have had the connections between the left and right cerebral hemispheres surgically severed in order to lessen severe epileptic seizures that did not respond to any other treatment. The majority of these persons are male and all of them have other brain injuries of various kinds. With these patients it is possible to test each hemisphere separately since the two hemispheres are no longer connected. While the operation is a drastic one, in fact, deficits in performance after the operation are not obvious and the operation serves to improve the condition of most patients.

The data at issue concern the difference, $p < .01$, found between three male and four female commissurotomy patients on their A/P ratio (Bogen, Dezure, Tenhouten, & Marsh, 1972). The A/P ratio is the ratio of appositional (right hemisphere) to propositional (left hemisphere) thinking. Appositionality was measured by performance on the Street figure-completion test and propositionality by the Similarities subtest of the Wechsler Adult Intelligence Scale. The commissurotomy patients used only their left hemispheres to do the tests. The significantly different A/P ration meant that compared to males, females were able to achieve a relatively better score on the Street test ( a supposedly right-hemisphere task) using only their left hemispheres. From this the authors concluded that appositionality (spatial function) was less lateralized in the females before the commissurotomy operation. That spatial function is less well lateralized in females than in males is not evidence that verbal function is less well lateralized.

At the present time data do not support a conclusion that females have more bilateral verbal representation than males. However, it is possible that, for example, among left handers, verbal bilaterality is more frequent or more pronounced in women.

Maximal cerebral organization hypothesis. The second proposition of the Levy hypothesis is that cerebral hemispheric specialization leads to maximal intellectual functioning, maximal presumable meaning best development of both verbal, analytic and spatial, gestalt functions. This hypothesis while plausible is not so plausible as Kinsbourne's (1974a, 1974b) description of interhemispheric interaction. Portrayals of interhemispheric relationships have underestimated the extent of bilateral interaction. In any case, Levy (1970, 1974) tested her hypothesis by comparing the relative verbal-spatial performance of right-and left-handed males. Since left-handed males more frequently have bilaterial verbal representation it was reasoned that this would interfere with maximum development of their spatial capacity. Therefore left-handed males should show lower spatial than verbal performance and lower spatial performance than right-handed males. This expectation was confirmed by Levy with a small, select male samnle, and by Miller (1971) (sex of subjects not reported). However, neither Briggs, Nebes, and Kinsbourne (in press) nor Sherman (Note 11) replicated this finding with larger, less select samples. The results of Newcombe and Radcliffe (1973) were also not consistent with the hypothesis.

McGlone and Davidson (1973) found a lower spatial performance of left-handers compared to right-handers with presummed right hemisphere dominance for linguistic function. This finding, as Levy (1976) notes, is not inconsistent with her hypothesis. McGlone and Davidson tested the Levy competition hypothesis by comparing subjects presumably using the same hemisphere, whether left or right, for both verbal and spatial function with those using either left or right hemisphere for verbal function and the other hemisphere for spatial function. The results do not show that spatial function is poorer among those presumably using the same hemisphere for both functions as opposed to those presumably using different hemispheres for each function. The results failed to support the Levy hypothesis. Results are unfortunately not reported separately by sex. The test is certainly a stringent one for the Levy competition hypothesis. A more cogent test would have been to compare, separately for each sex, spatial performance of persons with left hemisphere dominance for verbal function and right hemisphere dominance for spatial function with the performance of those with right hemisphere dominance for verbal and spatial function, for example.

Levy's rebuttal (1976) of McGlone and Davidson made many assumptions about unknowns when further empirical tests could presumably better resolve these questions. She analyzed the findings of McGlone and Davidson to show that the results are not inconsistent with a model of the inheritance of handedness and brain dominance (Levy & Naglaki, 1972). This model has not incorporated sex as a variable. However, Levy (1976) suggested that the hypothesized cerebral dominance gene is differentially expressed depending on the amount of sex hormones ranging from a little estrogen in XO persons, to more estrogen in XX persons to testosterone in XY persons.

This variation in presumed hormone strength would be paralleled by differences in brain lateralization and effectiveness of spatial function. The greater the hormone effect, the greater the lateralization, the better the spatial function. Testosterone is viewed as more effective than estrogen. As previously discussed, assumptions about the differential effects of "sex" hormones are hazardous indeed. Studies of XO persons have not reported testing them for extent of lateralization of function. It would be interesting to know if they are less lateralized for verbal or spatial function than XX or XY persons. However, it seems unlikely that Levy's line of reasoning will prove to be completely valid since some of the assumptions are not entirely congruent with data on persons of chromosome/hormone anomalies. First of all, XO persons can have as much natural testosterone from their adrenals as estrogens since they typically have no functional gonads of either sex. (Mosaics can be an exception.) Moreover, the XYY person who typically has normal and perhaps sometimes excess testosterone has poor spatial function. (See section discussing persons with sex chromosome/hormone anomalies and Table 1.)

Left-handed males like right-handed females. According to the Levy hypothesis, the cognitive patterning of right-handed females should be like that of left-handed males since both groups more frequently have bilateral verbal representation. Levy (1974) reported left-handed males having a markedly higher verbal than spatial performance, a pattern associat with females. Sherman (Note 11) did not find such a pattern in students, grades 9-11. Moreover, the intercorrelations among cognitive factors were very different for right-handed females and left-handed males. The correlations between math achievement and vocabulary, math achievement and spatial visualization and vocabulary and spatial visualization were all significantly different for right-handed females and left-handed males. These data strongly suggest that explanations of cognitive patterning associated with left-handedness will not suffice to explain cognitive patterning associated with the sexes.

Conclusion regarding Levy Hypothesis. This hypothesis lacks empirical support and cannot account for known data.

## Harshman and Remington Hypothesis

Harshman and Remington (Note 9) suggested that while females start out as more lateralized in language than males, this is only because females are on a faster maturational timetable. Males eventually surpass females and as adults are more lateralized for both verbal and spatial function than are females. This hypothesis assumes that lateralization is more favorable. Note also that Witelson (1976) and Rudel et al. (1974) report right hemisphere dominance for spatial function occurs earlier in males than in females, a finding which may necessitate revision of the Harshman and Remington model.

Lateralization as intellectually favorable. As already indicated it is not clear that being more fully lateralized is advantageous for intellectual functioning though there is some evidence to support this view for spatial function (Waber, 1975). Kinsbourne (1974a, 1974b) pointed out that strong lateralization is important for output, e.g., speech, but not necessarily for input nor internal behaviors (thinking?). Even Kinsbourne's view may be an oversimplification since the actuality of "normal" speech may depend on the complex co-ordination of both sides of the brain, each making itsown contribution to speech, the left hemisphere being responsible for control of the right side of the face and lips and enunciation of consonant sounds while the right side of the brain has control of the left side of the face and lips, resonance and vowel sounds (Smith, Note 13). There is evidence from commissurotomy patients that the right hemisphere does a better job with spatial, gestalt tasks while the left hemisphere does a better job with verbal, analytic tasks (Bogen et al., 1972; Levy, 1970). Hence type of lateralization as well as extent of lateralization may be important. Degree of lateralization as a general concept is probably meaningless.

Females verbally bilateral. As discussed in connection with the Levy hypothesis, there is no convincing evidence that females are more verbally bilateral than males. Harshman and Remington argue that the fact that more females than males may be right-handed does not refute their case for "normal" males. There are certainly valid aspects to this argument. On the other hand, data are not convincing that females have more bilateral verbal representation and some of the data cited to support this view are inappropriate. The striking difference between the cognitive patterning of right-handed females and left-handed males also argues against this view (Sherman, Note 11).

Females spatially bilateral. There are several converging lines of evidence indicating that for females more than males, the left hemisphere is more likely to be involved in performing spatial tasks. This, however, is not necessarily the same as saying that females are more spatially bilateral, nor can one infer that the brains of normal members of the two sexes are organized differently. It may be that females develop a preference for a left-brained, verbal, analytic approach. It is also possible that females may rely more on the left hemisphere while males may rely more on the right hemisphere for spatial function, both sexes being bilateral. The evidence of greater female left hemisphere involvement in spatial function consists of: (a) results of experiments in which electroshock was administered to a single hemisphere; (b) tachistoscopic and other studies; (c) studies of patients given temporal lobectomies; (d) studies of patients with naturally occurring brain lesions and (e) studies of commissurotomy patients.

Studying women before and after unilateral electroshock, females showed more deficit in performance on the Rod and Frame Test, a test of spatial perception, when the electroshock treatment was applied to the left hemisphere than when it was applied to the right hemisphere (Cohen, Berent & Silverman, 1973). In fact there was a paradoxical tendency for performance to improve after shock to the right hemisphere. Could this be the result of permitting total control by the left hemisphere, i.e. eliminating interfering competition form the right hemisphere (Kinsbourne,1974a, 1974b)? If so, this may provide some clues to the benefits of more complete lateralization though researchers would not ordinarily construe lateralization of spatial function to the left hemisphere to be a benefit since spatial function is "supposed" to be located in the right hemisphere. The evidence for the greater importance of th left hemisphere to female than to male spatial function would be stronger if the experiments were repeated on male subjects so that female results could be compared with male results.

Females more frequently than males have been found to be more accurate in enumerating dots tachistoscopically presented in the right visual field suggesting that the left hemisphere is more frequently of greater importance to spatial function in females than in males (Kimura, 1969, 1973; McGlone & Davidson, 1973).

An incidental finding of the Sherman (1974) study was that the performance of female subjects was less accurate when the frame of the Rod and Frame test was oriented to the subject's right rather than to the left while the opposite was true for males. For the second set of eight trials, the sex by frame orientation interaction was significant, $p < .05$, and the sum of chi-squares for the first three blocks of eight trails was significant, $p < .01$. This finding can be explained by the hypothesis that the left hemisphere is more involved in spatial function for females, while the right hemisphere is more involved for males. Rudel et al. (1974) found that girls develop more slowly in the performance of right-hemisphere-dependent tasks and that girls more than boys depend on left-hemisphere mediation for such tasks.

Also cited by some as evidence supporting the view that females are more bilateral in spatial function than males is the finding that both left- and right-handed women performed more poorly than left- and right-handed men respectively on a task of right-left discrimination, a spatial task (Bakan, 1974).

With temporal lobectomy patients, Lansdell (1962) reported that left lobectomy impaired judgment of design in females while right lobectomy improved their performance; the reverse was true for males. The finding that right lobectomy in females improved spatial performance is consistent with the finding from unilateral electroshock studies that female spatial performance was improved with shock to the right hemisphere. That is, for females, spatial performance improved without the <u>right</u> hemisphere contribution. For males, however, spatial performance improved without the left hemisphere contribution. Lansdell (1968) found that after left-sided temporal lobe surgery, females had lower scores than males on the Block Design and Object Assembly subtests of the Wechsler Intelligence Scale (spatial tests) while in the case of right-sided surgery, males had lower scores than females, $p < .05$. The temporal lobectomy data of Meyer and Jones (1957) analyzed in Sherman (1974) are consistent with Lansdell (1968). For left-sided surgery there was more spatial deficit for females than for males, $p < .10$; for right-sided surgery there was more spatial deficit for males than for females, p .05. Presumably the interaction effect would be statistically significant.

As already indicated, there are many inherent difficulties in drawing inferences from studies of <u>naturally</u> occuring lesions and many of the older studies did not test for sex-related differences and were not designed to evaluate such differences. Two recent studies already mentioned explicitly tested for sex-related differences (McGlone & Kertesz, 1973; McGlone, Note 12). Males with right cerebral damage tended to show greater impairment on the Block Design subtest of the Wechsler Adult Intelligence Scale (WAIS) than did females with right cerebral damage,

$p < .10$. However, females with left-sided damage did not show significantly greater impairment on the Block Deisgn subtest than males though findings are in that direction. Block Design scores correlated .63 with the scores on the Aphasia battery among the 22 females with left-sided damage compared to a correlation of .23 for the 32 males with left-sided damage. This sex-related difference in correlation is not statistically significant, but the direction of the finding suggests the greater participation of the left hemisphere in female spatial functioning (McGlone & Kertesz, 1973).

In another study by McGlone (Note 12), there were no significant differences for either sex in WAIS Performance IQ depending on whether the damage was left-sided or right-sided. In males, however, mean Verbal IQ was lower than Performance IQ for subjects with left-sided damage, $p < .01$, while mean Performance IQ was lower than Verbal IQ for right-sided damage, $p < .01$. No such differences were found for females. McGlone interprets these data to indicate greater laterality of function in males than in females. These data provide only slight support for the specific proposition that the left hemisphere is more involved in female than in male spatial functioning, (The brain injured persons consisted of both vascular and tumor cases; for the female tumor cases, those with left-hemisphere damage had a mean Verbal IQ higher than Performance IQ, a finding in the same direction as for right hemisphere damaged males, but the Verbal-Performance IQ discrepancy was in the opposite direction for vascular cases. Hence there is some slight support for the proposition of greater left hemisphere involvement in spatial function for females in part of the data.) On the whole variations in type of injury, time since injury and the impossibility of making more than an approximate guess in estimating prior intellectual functioning in order to match groups, make data from studies of naturally occurring brain lesions more suggestive than convincing.

The finding already discussed, from commissurotomy patients, that the Street-figure completion test, a spatial test, was performed better by the left hemisphere of female than male patients is, of course, striking evidence in support of the proposition that the female left hemisphere is more involved in spatial function than the male.

The evidence on the whole supports the view that for human females the left cerebral hemisphere is more involved in spatial functioning than is the case for males and that males rely more on their right hemisphere for spatial function than do females. But is this evidence that females have more <u>bilateral</u> spatial function? Not necessarily since it could be that both sexes use both hemispheres but that there are more females who rely on the left hemisphere especially. This would be consistent with evidence that left hemisphere dominance is established earlier in females than males for verbal function (Rudel et al., 1974; Witelson, 1976). That is to say, the sexes could be equally bilateral (or nearly so) but in different directions. The fact that male spatial performance improved after left temporal lobectomy suggests that this view may be more accurate.

Conclusion regarding Harshman and Remington hypothesis. Evidence of the greater bilaterality of verbal function in adult females is lacking and there is no clear evidence that lateralization is favorable the kind of lateralization is also important. Females may simply use their left hemispheres more for spatial tasks and males their right. This does not even necessarily make one sex more bilateral than the other. The Harshman and Remington hypothesis is not supported.

Bent Twig Hypothesis

This hypothesis (Sherman, 1967) began with the recognition that females, as a group (right-handeness assumed), have an early edge in verbal function which continued into adulthood at least for verbal fluency. About these observations there is little disagreement. It was hypothesized that the precocious verbal development of females caused them to develop a preference for a verbal approach, and consequently to neglect a non-verbal approach to problem solving (Sherman, 1967, 1971). This has been called the "bent twig" hypothesis and can rest on the law of primacy. Later indications that the left hemisphere is more involved in female than in male spatial function (Sherman, 1974) resulted in some refinements of the bent twig hypothesis. It would now appear that females not only have a preference for a verbal approach, but a preference for a left hemisphere approach. That is, they tend to rely on the left hemisphere not only for verbal function but for part of spatial function. Their verbal, left-brained approach is even more established by their verbal education and sex-role expectations. Presumably, a normal right-handed person using ~~the~~ left hemisphere to do a spatial problem approaches it in a verbal, analytic way, while a person using the right hemisphere would approach the task in a spatial, gestalt way. Those females with strong tendencies to use the left hemisphere for spatial tasks would do more poorly because the spatial, gestalt approach is better for spatial tasks (Bogen et al., 1972). Some additional support for the importance of verbal function (left-hemisphere) compared to spatial function (right-hemisphere) for females may be found in the factor analytic study of McCall (1955). He reported that verbal function was more important to female than to male intellectual performance. More involvement for females of verbal, analytic function in problem solving has been reported by Meyer (Note 14) and Schonberger (1976).

Conclusion regarding bent twig hypothesis. The bent twig hypothesis would appear to be more congruent with the evidence already reviewed regarding (a) the comparative development of cognitive skills in males and females; (b) sex-preferred modes of problem approach; (c) evidence from studies regarding localization of cerebral function; (d) evidence from studies showing earlier development of dominance of the left hemisphere for verbal, analytic function in females.

## Adolescence, A Second Point of Differentiation?

Waber (1976) studied the comparative verbal-spatial performance of early and late maturing boys and girls. Girls were selected from ages 10-13, boys from ages 13-16. The final sample of 80 included ten early and ten late maturing boys and girls at both age levels. Waber found that within individuals and regardless of sex, early maturers scored better on verbal than spatial tasks, and late maturers scored better on spatial than verbal tasks, $p < .01$. In the older group, but not the younger group, late maturers showed greater lateralization than early maturers. Waber argues that sex-related cognitive differences reflect differences in organization of cortical functions that are related to differential rates of physical maturation. Verbal performance was not related to maturation rate while spatial performance was, leading to the conclusion that sex-related differences in verbal and spatial skills "may have very different etiologies and cannot be explained by a common set of causes," p. 573.

Waber selected her early and late maturation groups from girls 10-13 and boys 13-16. While this procedure would be adequate if one could assume no differences in maturation rate for verbal and spatial skills from ages 10-16 for the two sexes, her own data show that there are some differences and these differences may have distorted some of the findings. Using just data for 13 year olds to equate for age, verbal-spatial difference scores were related to maturational rate, $p < .05$. Nonetheless, it would be well to repeat the study including fully mature persons. Another difficulty with the study is the fact that the relationship of lateralization and rate of maturation is with the absolute degree of lateralization without regard to direction, i.e., dominance of the left or right hemisphere for verbal sounds. While one supposes that generally the left hemisphere was dominant, interpretation of the results cannot be very clear cut. For example, since handedness apparently was not controlled, some of the relationships between degree of lateralization could be related to handedness. Petersen (in press) failed to replicate these results.

## Conclusions Regarding Laterality Hypotheses

Laterality explanations of the comparative cognitive skills in the two sexes are interesting, provocative, suggestive of innumerable research tactics and even some practical options. Many of the assumptions underlying these hypotheses have been shown to be questionable and existing data contradict many points. It is possible, however, that females (assuming right handed) have more left hemisphere involvement in spatial function than do males. The evidence that females have more right hemisphere involvement in verbal function than do males is much less impressive. The evidence rather suggests that the left hemisphere tends to be the sex preferred hemisphere for females.

That more females than males may be oververbalized and lean too heavily on left-hemisphere, verbal, analytic functioning to solve tasks that would benefit from more right hemisphere participation seems to be

a very viable hypothesis. This left-hemisphere preference may have a biological basis in the earlier maturation of females and specifically in the earlier development of linguistic facility and left-hemisphere dominance for verbal function in women.

Another point of possible biological influence may occur at adolescence when again the earlier maturational timetable of females means that more females than males mature early. Later maturation may favor right hemisphere development. Evidence on this point is not now sufficient to warrant any definite conclusion. Much of the biological basis for cognitive sexual differentiation would thus rest on maturational differences.

The ways in which maturational differences affect cognition may not be mechanistically biologic, but have to do with educational and cultural customs of timing. That is, introduction of subject material and various activities, sports, heavily depends on age, not maturational level, in our culture. It may be that these cultural timetables are better set up for some persons or groups of people than for others. The history of education has been primarily the history of male education. It is entirely possible that males more than females are better suited by the type and timing of educational experience they receive. The education females receive may not only be too verbal, lacking in spatial training, lacking in encouragement in the math-science areas, lacking in role models in math and science, but intoduction of education experiences may be inappropriately timed for more females than males.

Cognitive behaviors are very much under cultural influences and any biological influences have certainly been amplified in our culture. People have a tendency to think of biological influence as immutable and somehow sancrosanct, at least in some contexts. What is thought "natural" may be valued at some times, but certainly not at others. For example though it is not "natural" to read, reading is valued. Though it is "natural" to die, dying is not valued. There is also a tendency to give primacy to biological over cultural influence and to ignore the degree to which culture can shape biology itself. In recent years, people live longer than before and women can alter their biology so as not to produce children, to mention only a couple of examples. These examples represent intended human interventions into biology, but we do not know the extent to which our culture unwittingly influences biology. We know that cultural influence can affect handedness, a commonly selected index of brain laterality (Levy, 1974). Cultural influence can affect hemisphericity, a cultural preference for left-brained or right-brained thinking (Bogen et al., 1972). The nature of this preference for left-brained or right-brained thinking may have arisen because of the demands of the culture or the sex role though they may no longer be functional or they may not be functional for every task.

In regard to practical questions of research that would improve the achievement of women in mathematics, the most interesting question to pursue is the effect of right-brain training on spatial skill and mathematics performance in females and the effect of that training on sex-related differences in cognitive performance. Can women who use the left hemisphere for verbal tasks be trained to switch to their right hemisphere for solution of spatial tasks? Can techniques be developed to train that hemisphere? Would this be beneficial? Does the verbal education emphasis increase the suppression of right hemisphere thinking in women? Deeper and more philosophical questions, but perhaps of equal importance, have to do with acceptance of change in this aspect of women's role. Would right-brained, intuitive thinking be accepted from a woman? Would mathematical creativity in a woman be recognized (Helson, in press: Osen, 1974)?

Two research approaches that definitely do <u>not</u> seem worth pursuing are (a) studies of performance in persons with naturally occuring brain lesions. The difficulties of controlling all relevant variables and the absence of pre-post or adequate control information make these data unlikely to provide further elucidation. (b) Studies of cognitive factors related to handedness seem unlikely to be helpful. Handedness is a <u>very</u> indirect index of brain laterality and though handedness may be important as a flag to disturbances, its usefulness in theoretical illumination of questions of sex-related cognitive differences in highly questionable.

Other indications for research are tne need for more studies on biologically mature persons as well as broad based developmental studies and the need for investigators to include both sexes, to report and analyze the sex variable and to control for handedness, and history of handedness.

## Summary

It would be difficult to find a research area more characterized by shoddy work, overgeneralization, hasty conclusions, and unsupported speculations. This is particularly unfortunate since biological factors in sex-related cognitive differences in a research area with considerable social risk to the female group. It is also unfortunate because the poor quality of the research introduces confusion and retards the evolution of intelligent, well grounded opinions on this subject.

These observations themselves lead to some cogent questions. Why are researchers so willing to speculate about so little evidence? Are there any ways in which the quality of the work can be improved? Is there a need for guidelines for research in this area? Would researchers and editors benefit from a discussion of the technical, methodological, and value issues involved in this research? Does the fact that few women have positions of power and status in the academic/research community influence the questions asked and the answers obtained?

The hypothesis of the greater variability of intelligence in males is not well supported, particularly in regard to there being more exceedingly outstanding males. The important factor here probably has more to do with the fact that genius and intellectual creativity are not only not parts of the female role, but positively contraindicated. One wonders for example, if little girls would even be permitted to spend hours and hours working on mathematical problems, let alone be encouraged. In terms of practical implications for women, the studies indicate the meager opportunities afforded gifted women for the development and expression of their talent. Roberts (1945) is one who thought that there are indeed both more male defectives and male geniuses. He estimated that there is only one female genius for every two male geniuses. Even within the scope of limitations for the female sex suggested by such a theoretical stance, one finds much room for female development. It is touching indeed to note the concern of men like Roberts and Terman for the poor lot of gifted women. When the day arrives that even as many as on third of the geniuses are female, one can know that enormous strides have been made in the status of women. In light of this fact, it would seem far more advisable to direct research toward the discovery, development, acceptance, and expression of female talent than toward answering the evasive and methodologically slippery question of whether males show more variability in intelligence than females.

The gout hypothesis is unusual in that status and prestige have become attached to a disease state. The metabolic bases of thinking, ambition and effectance motivation are fascinating research questions. Equally fascinating are the questions of how attitudes, thoughts and emotions are translated into metabolic terms. Answers to these questions will not be quickly forthcoming. There is evidence that throughout much of the life cycle more males than females have high levels of serum urate acid. Whether this is a true effect or the product of uncontrolled factors such as activity level is not entirely certain. Whether sex-related differences in urate level is cause or effect of achievement-related behaviors and whether this is relevant to any sex-related differences in ambition that may be documented is not at all clear, though it is an exceedingly improbable source of significant variance. There is no evidence that serum uric acid level, in grossly normal ranges, is significantly correlated with intelligence. It appears highly unlikely that research in this area will increase our understanding of female intellectual development.

Thus far no credible hypothesis has been formulated which relates hormonal sex differences and differences between the sexes in cognitive patterning or in cognitive performance. The Broverman et al. (1968) theory is clearly unsupported. Within grossly normal ranges, effects of "sex" hormone variation on women's intellectual functioning have not been found. There is no evidence that differences in hormones can account for sex-related differences in cognitive patterning or performance. There is no reliable evidence that the amount and/or kind of prenatal, hormones especially masculinizing hormones, affect the organization of the human brain in terms of its cognitive functioning.

It is probable that girls and women would benefit from an increased understanding and acceptance of their bodily and hormonal functioning. This could be a source of improved functioning for a subgroup of women whose knowledge and acceptance of bodily functioning continues to be affected by the cultural repression of the female body and sexuality characteristic of 19th century American thinking (Smith-Rosenberg & Rosenberg, 1973). Topics in this area might well be a fruitful area for research, curriculum development and change within the educational system. Earlier, continuous and much more thorough education in bodily functioning specifically including hormonal changes, menstruation, pregnancy, childbirth, menopause, and health related issues such as venereal disease, bladder infections, signs of pregnancy, diseases of women, e.g., breast cancer, needs to be embedded in the educational curriculum. "Sexuality" has been introduced in recent years in the school curriculum, but especially for women the teaching of sexuality is not adequately situated in the overall context of bodily functioning. The introduction of these topics could also be made in such a way as to stimulate the interest of women in science and mathematics, e.g. use of graphs, sample statistical concepts.

The hypotheses of the recessive, X-linked inheritance of spatial and mathematical abilities are not confirmed. There appears to be no further need for research in this area.

Questions of the relationship of brain organization to effective intellectual functioning and to sex-related differences in cognition represent a hotly pursued and rapidly changing area of knowledge. At this point it appears that there may be some relationship between the type and degree of brain lateralization, sex-related differences in cognition, and timing of maturation with two possible points of differentiation - preschool years and adolescence. The most reliable aspects of available data suggest the hypothesis that females, partly as a consequence of earlier maturation and partly as a consequence of cultural influences, develop a reliance on verbal, analytic, left hemisphere approaches to problem solving and tend to use this approach for tasks that males might solve using a spatial, gestalt, right hemisphere approach.

Verification of this hypothesis, ruling out alternative hypotheses, understanding the role of timing, quantity and type of hormonal, maturational, experiential, and cultural contributions to these phenomena encompass an enormous range of research questions. If the hypothesis is correct, there are likewise a great number of questions raised. How can one train the right side of the brain? What are the difficulties and benefits of such training? Should the educational curriculum be revised to include greater emphasis on such training? When should such training be introduced? Who can most benefit from such training?

## Remedial, Therapeutic versus Developmental Strategies

Plans for remedial or compensatory education for women, while well intended, place the emphasis on deficit and may unwittingly contribute to the idea that females are in some way defective and inferior. The therapeutic approach is even worse in this regard. In this viewpoint women are regarded as not pursuing mathematics because of emotional problems. It is doubtful that emotional problems impede female development in mathematics any more than in male development. Girls and women are much more likely to admit being distressed and to display distress (Sherman, 1971). Casting research and programs into a developmental frame with an emphasis on providing for maximal development of all children in the context of special individual and group needs facilitates more health self concepts and group concepts. A broadly based developmental approach has much more prospect of preventing problems and providing lasting benefit than intervention programs that may prove to be only of cosmetic value.

## Theoretical Background

The recommendations for research and development of educational technology stem from a set of understandings about the development of cognitive sex-related differences. This theory is by no means complete and the argument and evidence to support it in detail cannot be presented within the scope of this review though they have in part been presented elsewhere (Sherman, 1967, 1971, 1974, 1976a, 1976b; Fenema & Sherman, Note 1, Note 2; Sherman & Fennema, Note 17). This theory is based on conclusions drawn from an overall understanding of the research literature and theory in the areas of cognitive development, psychology of women, sex roles, as well as the specific area of the development of mathematical skills.

The differences between the sexes in intellectual functioning are minor and could not possibly account for the few numbers of women in mathematics and mathematics related fields. The extent and kind of biological factors that might be involved in creating these differences is a matter of scientific controversy as the preceding review attests. By far the most important biological factors may ultimately prove to be the larger size and physical strength of the adult male, and the differences between the sexes created by the reproductive and nurturing activities of females. These indisputable biological factors have influenced sex-role training and expectations in ways which we still but dimly understand.

The possible role of these biological factors in affecting sex role can be seen most clearly in the development of spatial skill which leads indirectly to those activities we more commonly describe as mathematics. The association of female with the hearth, male with the out-of-doors is a persistent and pervasive generalization, extending from primitive societies to our own. Hunting has been a strong male domain. There are good biological reasons for this

nearly universal custom. Certainly a single woman away from home base runs a greater risk of being raped and/or kidnapped than a male. This problem stil limits the mobility of women. Women pregnant or with small children could not traverse the distances nor cope with the hardships of being away from the home base so well as men. Ninety percent of sexually active females in the reproductive ages, without birth control and not breast feeding, become pregnant each year. The reproductive function of women was very salient in earlier times and under more primitive conditions. Women had many pregnancies, lost many babies and often died in childbirth. Many fewer women survived the childbearing years. Menstruating women constituted a hazard and a handicap in a hunting party and would be at risk as a single hunter. Wild animals can smell blood at great distances. Most of the persons recently killed by unprovoked bear attacks in the American Rocky Mountains have been menstruating women.

These biological factors, very salient in earlier times and under primitive conditions, have had much to do with the development of sex-role expectations. Major activities away from home base have been assigned to males though females have left the home to gather food and herbs and, in agricultural societies, to tend the fields; the division of labor has not been complete, but it has been substantial.

The assignment of the major role in hunting and other activities involving the traverse of space to males has had profound implications for the development of sex-role socialization. All matter of activities involving aiming, visualizing direction, estimating space, and symbolizing space and spatial relations (the essence of mathematics) has been an integral part of male socialization experiences. Even the spatial task of disembedding, measured by such tests as the Embedded Figures Test or Hidden Figures, is not so remote from sex-role considerations as it might seem. The perception of an animal as distinct from its surroundings is a classic task of visuo-spatial disembedding, and one involving great skill and practice. Disembedding as well as aiming and finding one's way are essential to good hunting. Many activities sex-typed as male serve to develop spatial skill. These activities are represented in the toys and games given to children and which they are encouraged to play with, directly and indirectly, by their own knowledge of their future adult roles. Early differences in play activities are later supplemented by a host of other socialization experiences including divergent channeling of the sexes into particular courses in school, e.g., drafting.

When relevant factors are controlled, differences between the sexes in mathematics performance are minimal or nonexistent (Fennema & Sherman, Note 1, Note 2). However, when these factors are not controlled, i.e., when "natural" groups of males and females are compared, sharp sex discrepancies in mathematics performance have been found,

e.g., the National Assessment of Educational Progress (Mullis, Note 18). This sex discrepancy is seen to stem fundamentally from a pervasive network of sex-role influences.

## Implications of the Theoretical Orientation

The primary need is to place mathematics achievement within the scope of behaviors appropriate for girls and women and to provide through formal instruction those necessary learning experiences which for boys flow mostly from their informal experiences. The details of the impact behaviors that are performed or not performed in the school environment are certainly not entirely known, but some intelligent guesses can be made about needed changes.

Systems approach. To be effective any program must involve all the important systems affecting the learning of mathematics including parents, teachers and counselors. There is evidence that parents and teachers are important to math achievement, that their attitudes toward boys as learners of mathematics are perceived by students as more favorable, and that these perceptions differentiate high school students who continue or do not continue in mathematics (Fennema & Sherman, Note 2; Sherman & Fennema, Note 17; Sherman, Note 19). Changes introduced will need to include congruent changes in behaviors of parents, teachers and counselors.

The self system is intimately tied to sex-role influences. Confidence in doing mathematics is an important variable in mathematics achievement and the willingness to continue to study mathematics (Fennema & Sherman, Note 2; Sherman & Fennema, Note 17). Confidence in doing mathematics may be enhanced by perception of its gender appropriateness, and by perception of the gender appropriateness of being competent in general. Any program or instituted changes will need to monitor the math confidence of girls and women to insure that the introduced changes are having the desired effect.

One aspect of the development of a self system perceiving mathematics as congruent with the female gender is exposure to successful models, for example, through female teachers skilled in math and science, especially at advanced levels of training. Modeling can also be facilitated by materials about women who have achieved in the math/science areas. Modeling of only slightly older peers who model successful coping might prove especially effective. Male teachers and counselors have a role to play in letting girls and women feel that they accept, encourage, and enjoy intellectual development, competence and mathematics achievement in girls and women.

Not only does each relevant system need explicit attention, but these different systems need attention extending over the entire educational time span. While important intellectual developments occur in the preschool years, there is a need for greater attention to the school years and later, including transition from school to work or other activities (Sherman & Denmark, in press).

The greatest research need, however, is for broad based, programatic, developmental research, which can provide the knowledge base for maximal development of cognitive functioning in the female sex. Most of our knowledge has been based on male subjects and the history of educational experience has probably been unwittingly aimed toward the maximal development of males. It is doubtful that anything less than such a broad based approach will provide the necessary knowledge.

## Recommendations[1]

A major purpose of this review has been to generate recommendations for research and development of educational technology to improve the quality of mathematics achievement in women and to increase their participation in activities, courses and ultimately in jobs and careers that involve mathematics or require considerable mathematical knowledge.

### Two Caveats

In some instances the need for mastery of advanced mathematics is spurious. Required mastery of mathematics not necessary to the performance of the job serves wittingly or unwittingly to exclude many women (and minority persons). Examples of this sort might be entrance exams for law school and medical school. These exams often contain questions involving advanced mathematics, yet advanced mathematics is not used in the daily work of most physicians and lawyers. Changing entrance tests and curriculum which require mastery of mathematics not necessary for job performance is a controversial step open to the criticism of lowering standards. Nonetheless, these questions should be considered and objectively evaluated.

Jobs which involve use of advanced mathematics (e.g., knowledge of differential equations) are not very common. For this reason, it would seem best to emphasize development of fundamental mathematical knowledge in a greater number of girls and women and to a lesser degree, to focus directly on increasing knowledge of very advanced mathematics in women. Because of the tendency for women to be nearly excluded from the very high status jobs, however, it is probably unwise to neglect this aspect entirely.

[1] Thanks are due to Professor K. U. Smith for his critical evaluation of this paper, particularly the Recommendations section. The proposal for a broad-based economics curriculum is derived from his work and ideas.

Destereotyping of mathematics. The preceding discussion has outlined some of the systems and variables involved, but the main problem of how to destereotype mathematics and how to provide, in the formal curriculum, for those experiences which boys normally acquire informally has not been touched upon. The latter involves mainly spatial skills, but the recent availability of hand calculators and small computers may prove another source of sex discrepancy. These tools of mathematics appear to be finding their way increasingly to males more than females.

A main thrust will need to be the development of curriculum of fundamental usefulness and inherently less stereotyped. The essential matter is to produce material involving mathematics in areas of strong female interest. To some extent this can be done by much stronger scientific and mathematical involvement in courses such as health and in new courses and modules involving female bodily functioning through the life cycle and child care. More intensive curriculum development in these areas would not only facilitate concomitant interest in mathematics, but would have broad social importance and application to health development for all Americans.

An area of curriculum development which lends itself more naturally to mathematics is economics. Knowledge in this area would be appealing to women and the curriculum developed would have major importance for other groups as well. A broad based program of grade school, middle school, high school and university training in money, financial and economic processes is proposed. At every grade level, courses would be developed, based on practical projects in money exchange: personal buying and selling; use of processing of checks, money order, credit cards; methods of saving; allowances; taxes and income taxes; wages and salaries; hospital, accident and home insurance; buying and selling homes; real estate; annuities; investments in stocks and bonds; business financial operations; home budgeting; filling out income tax forms. Quantitative and mathematical methods could be taught in each of these courses. These courses would also be used to introduce the new mathematical tools of calculators and small computers. Use of material and courses with greater female interest will help improve the perceived usefulness of mathematics which has been demonstrated to be an important variable (Fenema & Sherman, Note 2; Sherman & Fennema, Note 17).

Spatial training. Even though girls are now permitted to take courses such as drafting that were previously closed to them and even though there is now a tendency to enroll both boys and girls in industrial arts classes before the high school level, there remain informal discrepancies between boys and girls in their spatial training and in formal courses at the high school and college levels. It is difficult to know the whole range of informal activities that could be relevant to the development of spatial skills though the list includes blocks, construction kits, model building, direction finding, chess, and sports. Some awareness of the broader educational implications of these activities would be helpful and they could be promoted for girls.

More spatial and graphic exercises need to be introducted into the formal training in mathematics and there is a need to develop a programmed, largely self-administered, spatial training package. The availability of this package would provide a tool for teachers wishing to improve the skill of any one student. Such a package will be useful for many students.

While it is possible to proceed with such a kit on the basis of present knowledge, there is a great deal that is not known about development of spatial visualization especially in females. Two of the most acute questions are when to introduce such training and what sort it should be.

At the present time the literature on sex-related differences in spatial development is puzzling since there is some evidence that in the preschool years girls perform better than boys on spatial tasks (Coates, 1974); however, boys have been found to establish dominance of the right hemisphere for spatial tasks earlier than girls (Rudel et al., 1974; Witelson, 1976). If males have earlier right hemisphere dominance for spatial task and spatial function is a right hemisphere task, one would not expect girls to do better than boys. If these findings are replicated and valid, one possible resolution might be that girls achieve better earlier using the left hemisphere while later boys are able to do better using the right hemisphere. If this hypothesis is valid it may be that not only spatial training for girls is needed but <u>right hemisphere</u> spatial training.

<u>Esprit de corps.</u> Math-science clubs sponsored by the schools or by private organizations could prove a useful device for promoting interest, creative fun with math-science activities and a firmer sense of the appropriateness of these activities. Open discussion of sex role issues could occur more easily in such a setting and provide a mode for the development of leadership and team skills in an atmosphere normatively designed to allow girls to feel good about participation in mathematics and science. Leadership and programming of such clubs would need to be well thought out. The ratio of girls and boys would need to be equal or weighted in favor of girls, or even entirely female.

The strategies discussed cover all of the important variables found to be involved in creating sex-related differences in mathematics achievement: spatial visualization, stereotyping of mathematics as a male domain; perceived attitudes of parents, teachers; confidence in one's ability to do mathematics and perceived usefulness of mathematics. Not directly dealt with has been fear of success in mathematics. This variable was captured in our research and appears to be relevant, but it showed socio-economic variations and did not correlate very highly with mathematics achievement. Presumably destereotyping will eliminate negative factors from this source as well.

## Research and Development Needs

The following list sets out in brief form a summary of research and development in the area of developing female mathematical skill and participation. Emphasis was placed particularly on projects with nation-wide possibilities and projects that would not only benefit women, bu others as well.

### Research

Increased understanding of the development of spatial visualization in girls and the role of the two hemispheres of the brain.

Improved understanding of the factors allowing girls and women to pursue math-science courses and job plans successfully.

### Development

Techniques and materials for use in reducing sex-stereotyping behaviors in teachers

Techniques and materials for use in reducing sex-stereotyping behaviors in counselors

Materials for distribution to parents that will help facilitate their encouragement of girls in mathematics

Materials demonstrating successful coping of young women in mathematics and math-related fields

Self-instructional programmed packages for increasing spatial visualization skill, including a high school and college levels

Texts with mathematical problems, examples and materials more oriented to women

New course content oriented around female bodily functioning and incorporating materials to stimulate interest in mathematics and science

New courses teaching economics with special design to develop girls' mathematics

Math-science clubs designed especially to promote development in a supportive, destereotyped atmosphere.

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